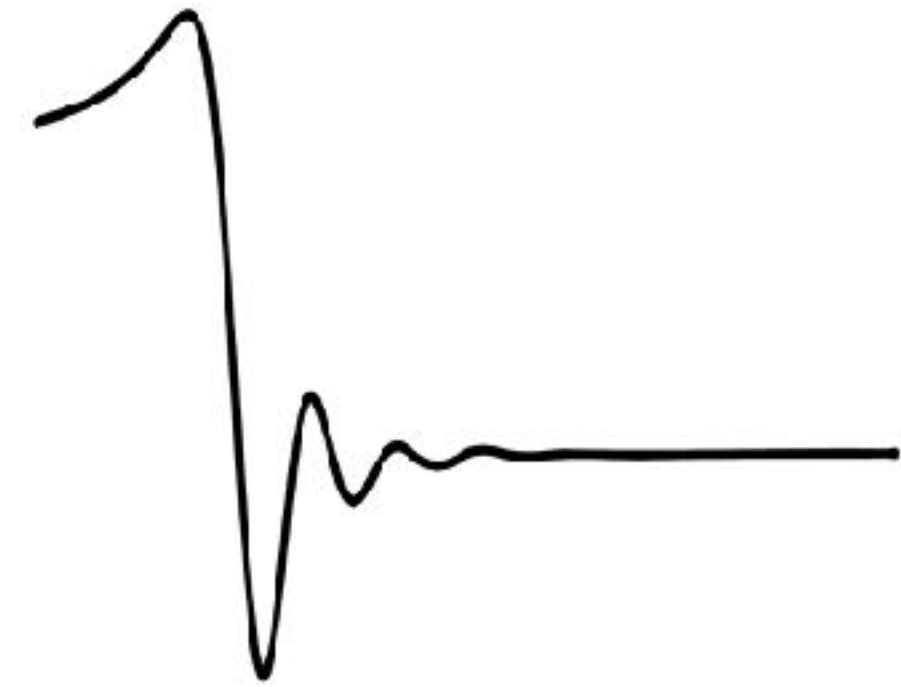


Gravitational wave observations of black hole mimickers: where do we look, and what do we look for

@ Black Hole Mimickers: From Theory to Observation

March 3, 2025

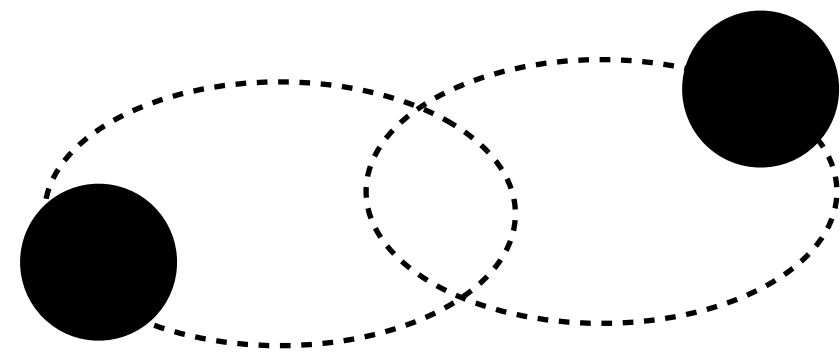


Andrea Maselli

Where do we look for signatures

Different families of sources with different orbital setups, to potentially observe BH mimickers

- (some) common observables but: different approaches, simplifications, waveform models



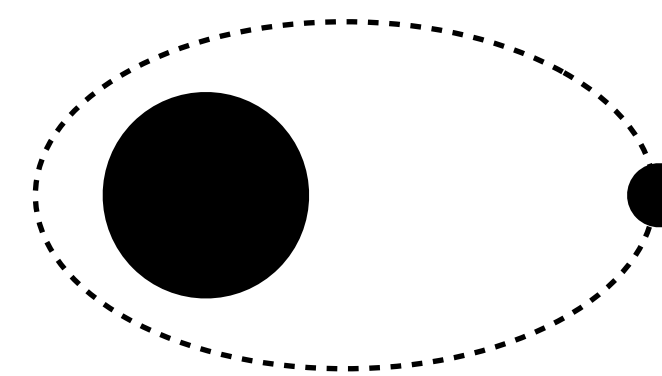
Comparable-mass binaries

post-Newtonian approach (inspiral)

Many observables and (some) waveform models already in “place”

Numerical Simulations

*lessons from analogous sciences cases
(neutron stars)*



asymmetric binaries

Self Force theory

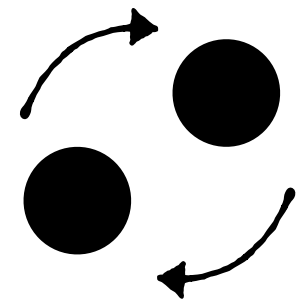
Simplifications due to mass asymmetry



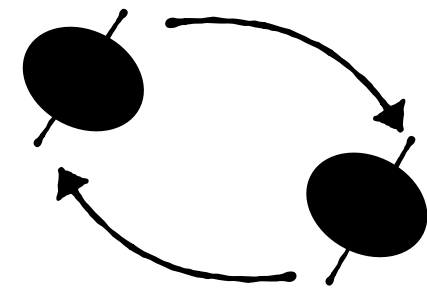
*moving to the
“microphysics” of
the model*

Where do we look for signatures

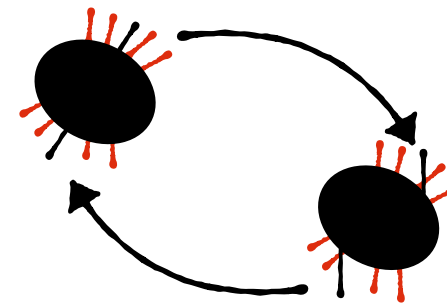
Through comparable-mass signals



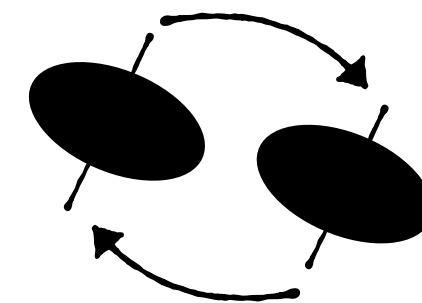
point particle



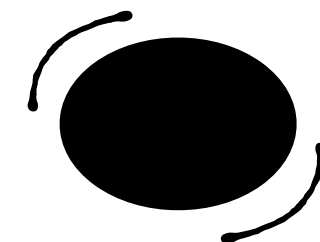
spin deformations



tidal heating



tidal effects



post-Newtonian relevance $(v/c)^n$



part of the “game” is already in place

What do we look for

Tidal interactions leave the footprint of the compact object structure on the GW signal

[T. Hinderer, 2008; T. Binnington & E. Poisson, 2009; T. Damour & A. Nagar, 200]

- Deformation properties of a compact object encoded within a set of Love numbers

$$\text{body's quadrupole} \longleftarrow Q_{ij} = \frac{2}{3} k_2 R^5 \mathcal{E}_{ij} = \lambda \mathcal{E}_{ij} \longrightarrow \text{external tidal field}$$

neutron stars

$$M_{\text{NS}}/R_{\text{NS}} \in [0.1 - 0.2]$$

$$\lambda \neq 0 \quad \lambda \sim \mathcal{O}(10^4)$$

BHs (in GR)

$$\lambda = 0$$

Schwarzschild
Kerr

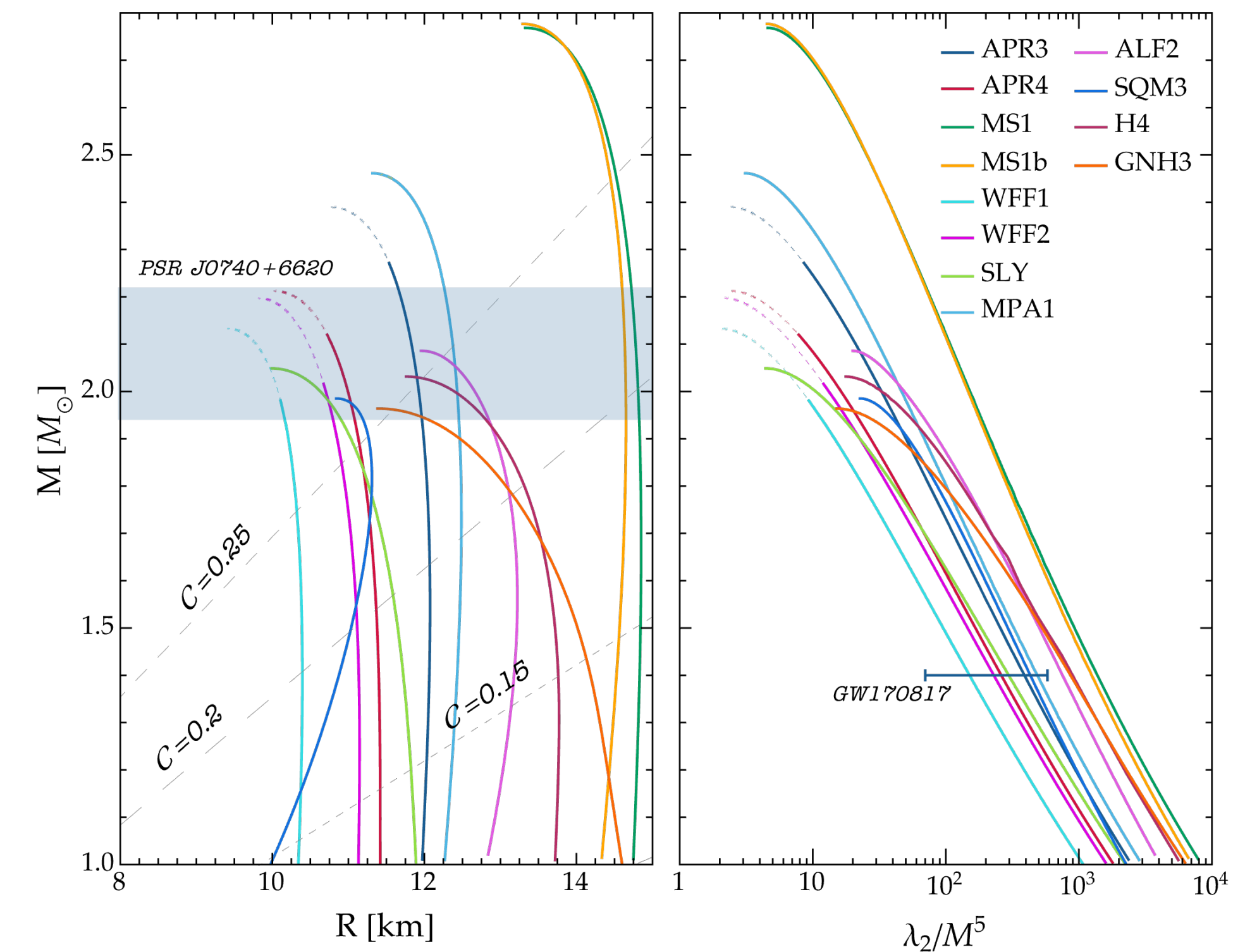
[P. Landry & E. Poisson, 2015; P. Pani + incl. A. M., 2015;
N. Gürlebeck, 2015; A. Le Tiec +, 2021]

BHs non vacuum/
beyondGR
ECOs

$$\lambda \neq 0$$

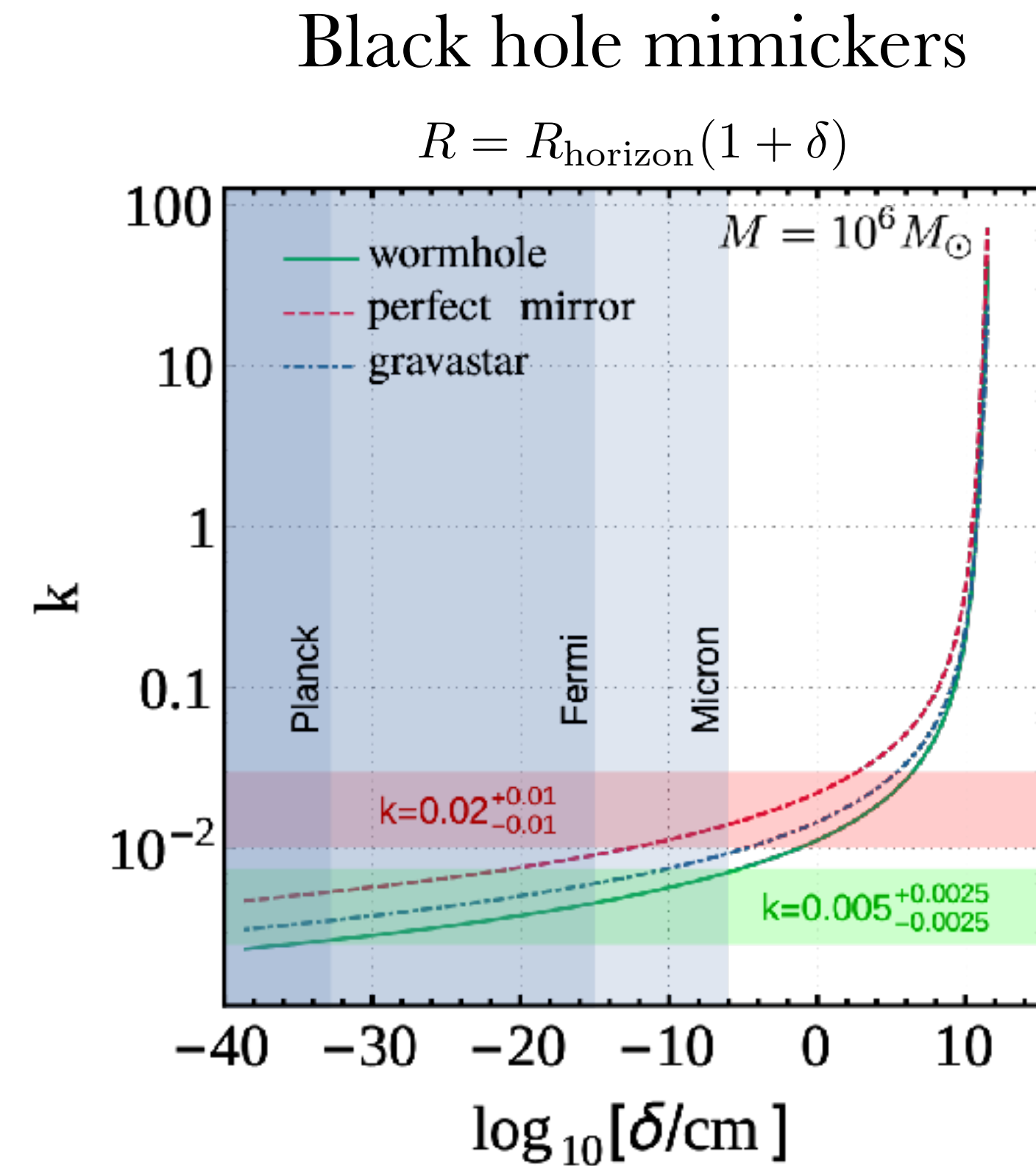
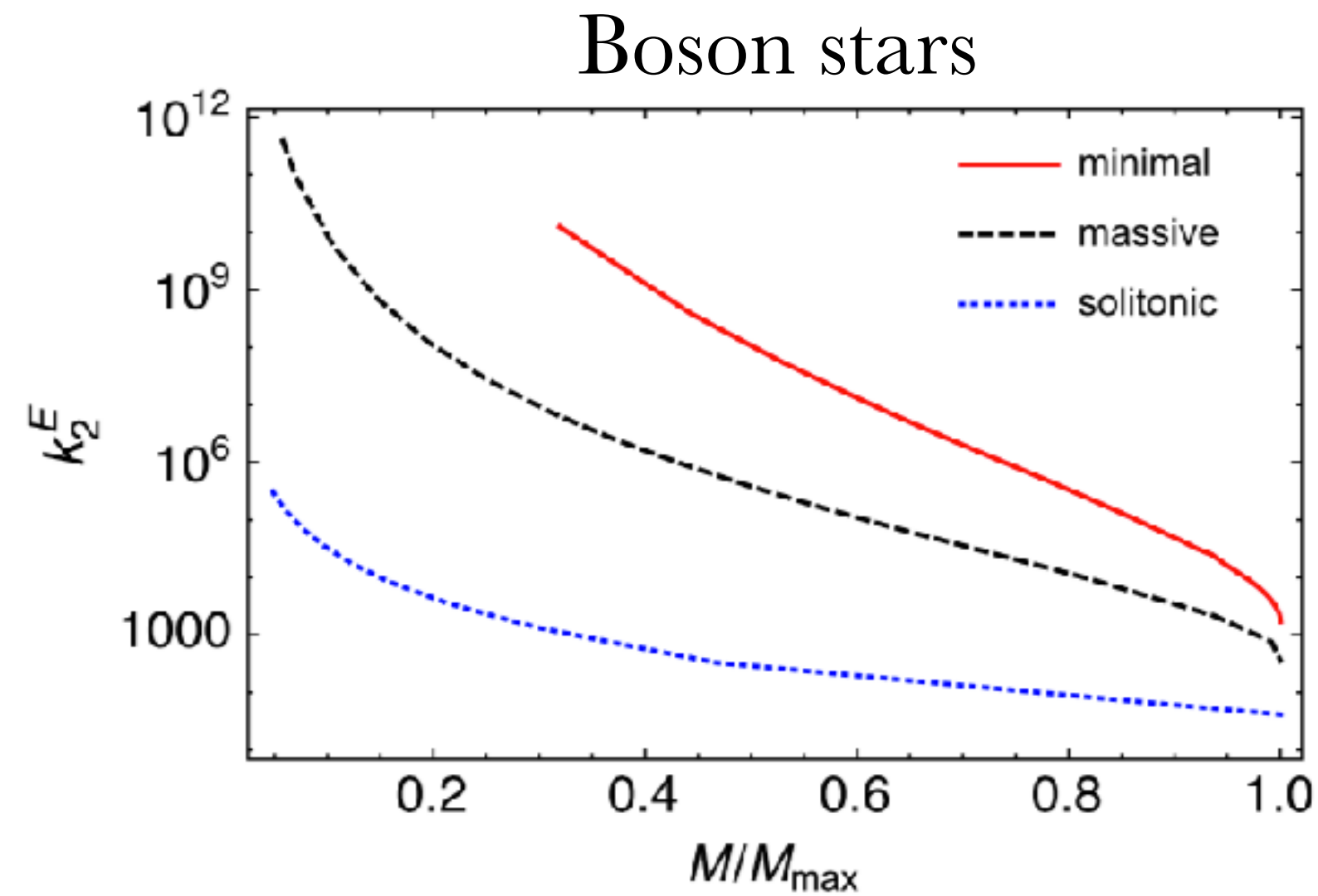
*generic tests of
gravity*

[V. Cardoso + incl. A. M., 2017; A. Maselli +, 2017]



Exotic Love numbers

[V. Cardoso + incl. A. M., 2017]



- Behaviour for BS in qualitative agreement with the neutron star case
- For mimickers Love numbers show a logarithmic dependence on δ

————→ the Love number enters the waveform

What do we look for

Multipolar structure of Kerr black holes benefits from axial/equatorial symmetry [R. O. Hansen, 1974]

$$M_\ell^{\text{BH}} + iS_\ell^{\text{BH}} = M^{\ell+1}(i\chi)^\ell$$

$$M = M_{00} \quad J = S_{10} \quad M_\ell = M_{\ell 0} \quad S_\ell = S_{\ell 0}$$

- Objects different from BHs can violate this relation and induce generic deviations [G. Raposo & P. Pani 2020; L. Bena & D.R. Mayrrson, 2020; M. Bianchi +, 2021; P. Cunha + 2022; N. Loutrel + inc. A.M, 2023, 2022]

$$M_{\ell m} = M_\ell^{\text{BH}} + \delta M_{\ell m} \quad S_{\ell m} = S_\ell^{\text{BH}} + \delta S_{\ell m}$$

- $\delta M_{\ell m}, \delta S_{\ell m}$ depend on the structure of the compact object
- The leading contribution to the waveform is given by the mass moment $M_{20} = -M^3\chi^2$ at the 2pN order $(v/c)^2$
 - agnostic way to parametrize deviations from Kerr $M_{20} = -\kappa M^3\chi^2$ and $\kappa = 1 + \delta\kappa$

A coherent waveform model

[M. Vaglio + inc. A.M, 2023]

A waveform beyond minimal modelling (for inspiral) $\tilde{h}(f, \theta) = \mathcal{A}(f, \theta)e^{-i\varphi(f, \theta)}$

- post-Newtonian expanded waveform with finite size effects, $x = (\pi mf)^{2/3}$

$$\varphi = 2\pi f t_c - \varphi_c - \pi/4 + \frac{3}{128\eta x^{5/2}} \times \left(\underbrace{\sum_{i=0}^7 \varphi_{\text{pp}, i/2} x^{i/2}}_{\text{point-particle}} + \underbrace{\varphi_{\kappa, 2} x^2 + \varphi_{\kappa, 3} x^3}_{\text{quadrupole}} + \underbrace{\varphi_{\text{T}, 5} x^5 + \varphi_{\text{T}, 6} x^6}_{\text{tidal}} \right)$$

$$\varphi_{\text{T}, 5} = -12[(1 + 7\eta - 31\eta^2)(\Lambda_1 + \Lambda_2) + \delta(1 + 9\eta - 11\eta^2)(\Lambda_1 - \Lambda_2)]$$

$$\varphi_{\kappa, 2} = -\frac{50}{m^2}(\kappa_1 m_1^2 \chi_1^2 + \kappa_2 m_2^2 \chi_2^2)$$

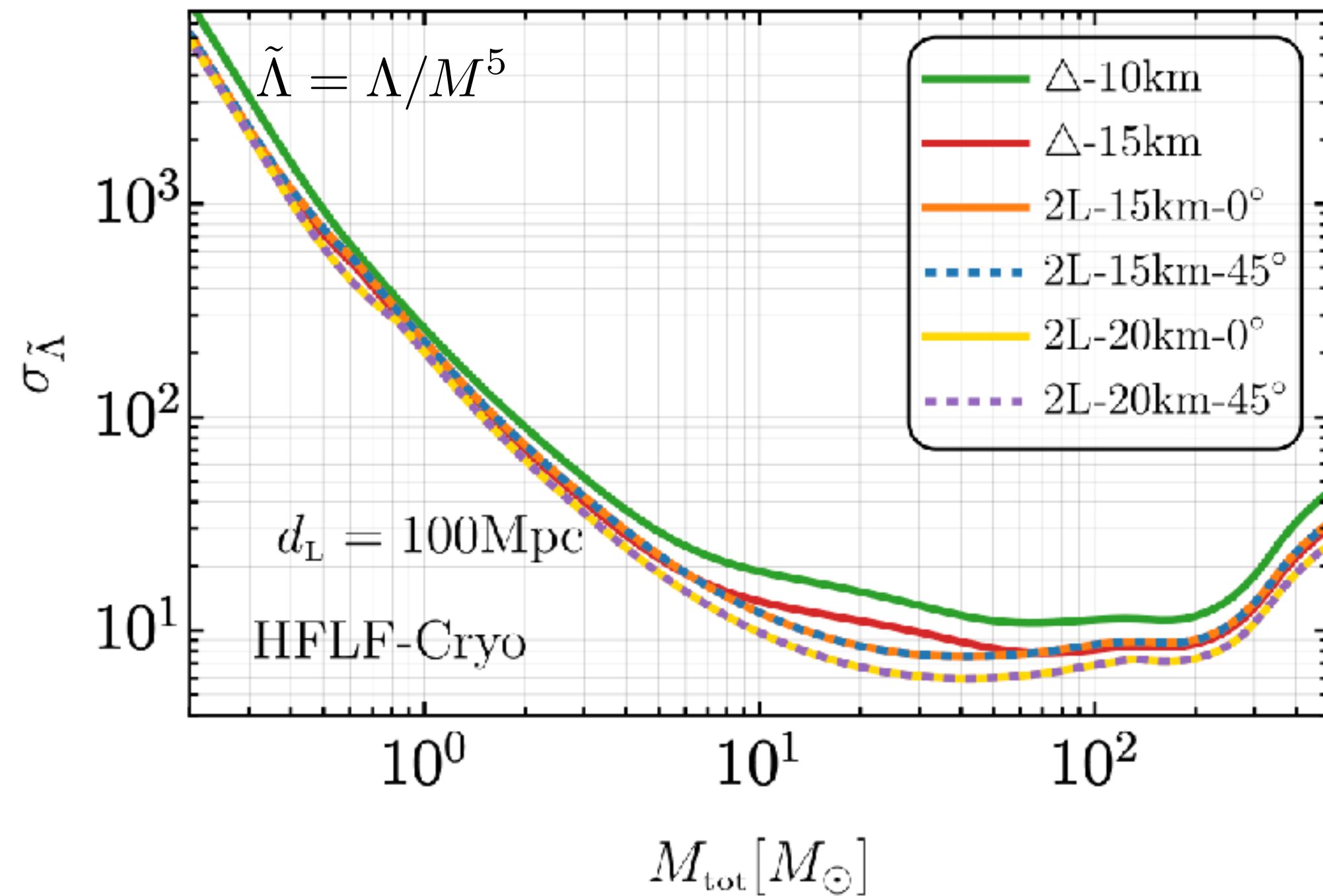
- introduce Λ_i, κ_i (or combination of) as independent single or multiple parameters, and constrain deviations from zero

$$\theta = \{\mathcal{M}, \eta, \chi_1, \chi_2, \kappa_i, \Lambda_i, d_L, t_c, \phi_c\}$$

Forecasts with ET

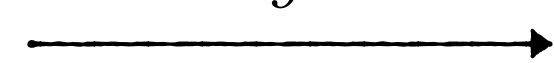
[M. Branchesi +, 2023]

Constraints on the tidal deformability and quadrupole deviations from ET observations



$$\theta = \{\mathcal{M}, \eta, \chi_1, \chi_2, \Lambda_i, d_L, t_c, \phi_c\}$$

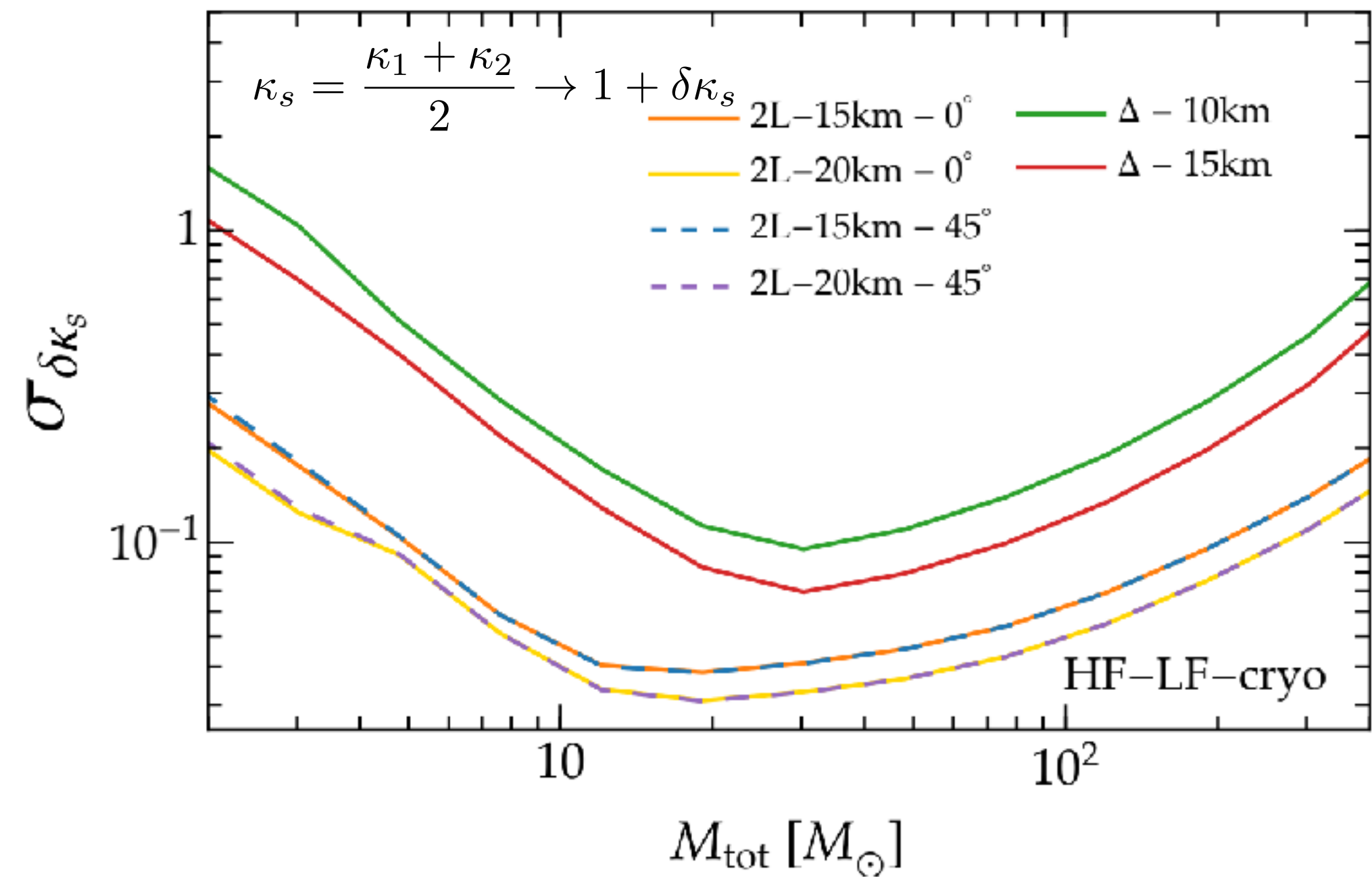
as a reference



NS mass

$$\sigma_{\tilde{\Lambda}} \sim \mathcal{O}(10)$$

$$\sigma_{\tilde{\Lambda}}^{\text{GW170817}} \sim \mathcal{O}(100)$$



$$\theta = \{\mathcal{M}, \eta, \chi_1, \chi_2, \kappa_i, d_L, t_c, \phi_c\}$$



$$\delta\kappa_s \sim 10^{-1} - 10^{-2}$$

$$\delta\kappa_s^{\text{GW151226}} \lesssim 10 - 100$$

[N. Khrishnendu +, 2019]

A coherent waveform model

[M. Vaglio + inc. A.M, 2023]

Or...introduce physics information to reduce the volume of the model

$$\mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\phi_{,\mu}^*\phi_{,\nu} - \frac{1}{2}\mu^2|\phi|^2 - \frac{1}{4}\sigma|\phi|^4$$

- In the limit $\sigma \gg \mu^2$ equilibrium solutions can be very compact
- Properties of equilibrium configurations depend only on the ratio $M_B = \sqrt{\sigma}/\mu^2$

Learning from the neutron star case, this allows to map *macroscopic* observables to *microscopic* fundamental parameters

[N. Sennet + 2017, ; C. Adams + 2023]

- Semi-analytic expression relating the static tidal deformability with $\beta = M/M_B$

$$\log_{10} \Lambda = \sum_{k=0}^4 \alpha_k \beta^k$$

- use this relation for spinning BS (but...[G. Castro + inc. A.M, 2020])

A coherent waveform model

● Quadrupole moment in terms of β and the dimensionless spin χ , $\kappa_2 = \kappa_2(\beta, \chi)$ [M. Vaglio + inc. A.M, 2022]

● Interpolation of numerical data obtained from multipolar structure of spinning boson stars

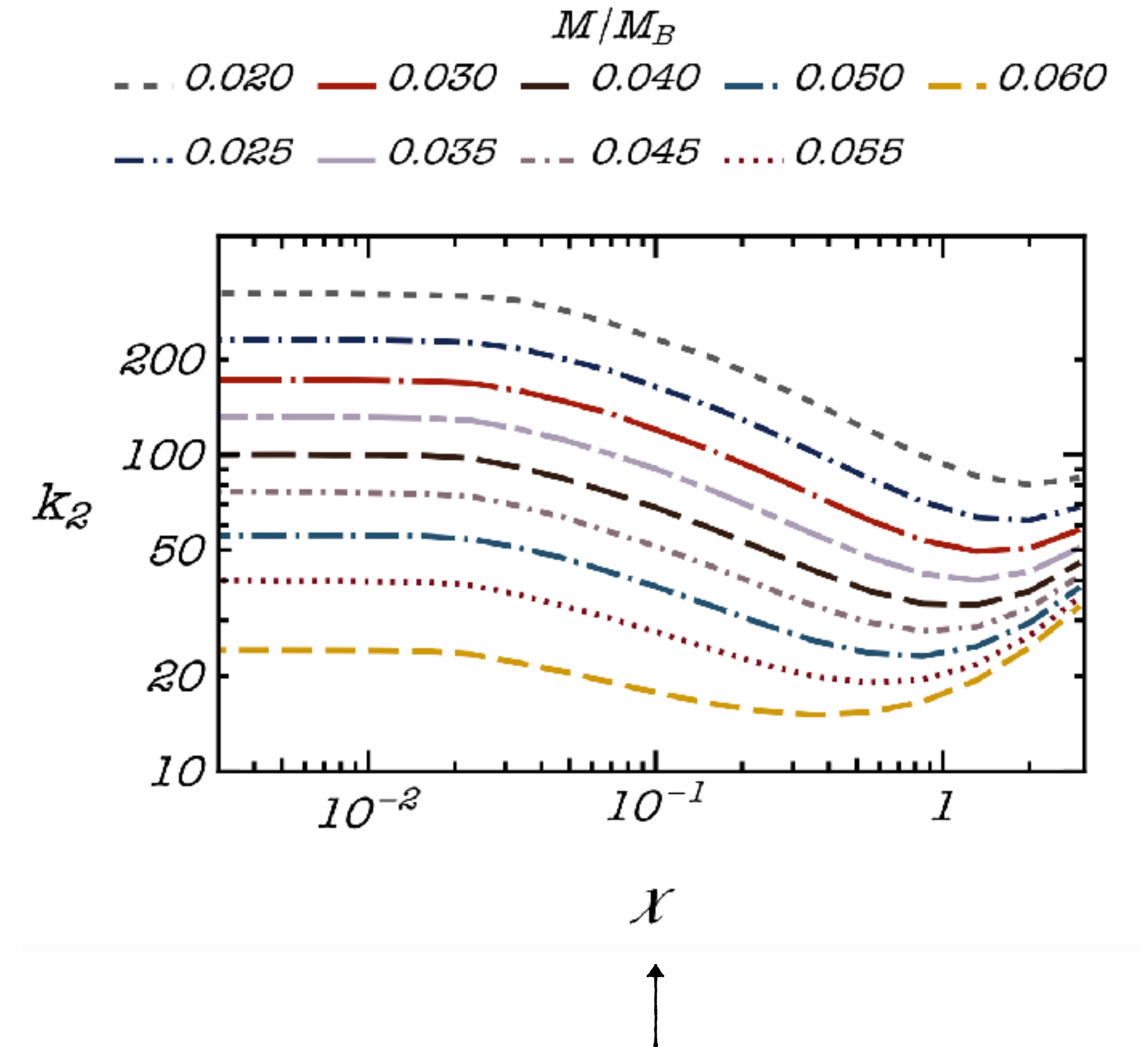
● $M_2 = -\kappa_2(\chi, \beta)\chi^2 M^3$, with $\kappa_2^{\text{BH}} = 1$

→ universal relations among
BS parameters

● Deviations from BH evolution encoded within a single parameter

$$\theta = \{\mathcal{M}, \eta, \chi_1, \chi_2, M_B, d_L, t_c, \phi_c\}$$

Can we directly constrain M_B from GW observations ?



$\kappa_2 \sim \text{const}$ only at low spin
& has a minimum

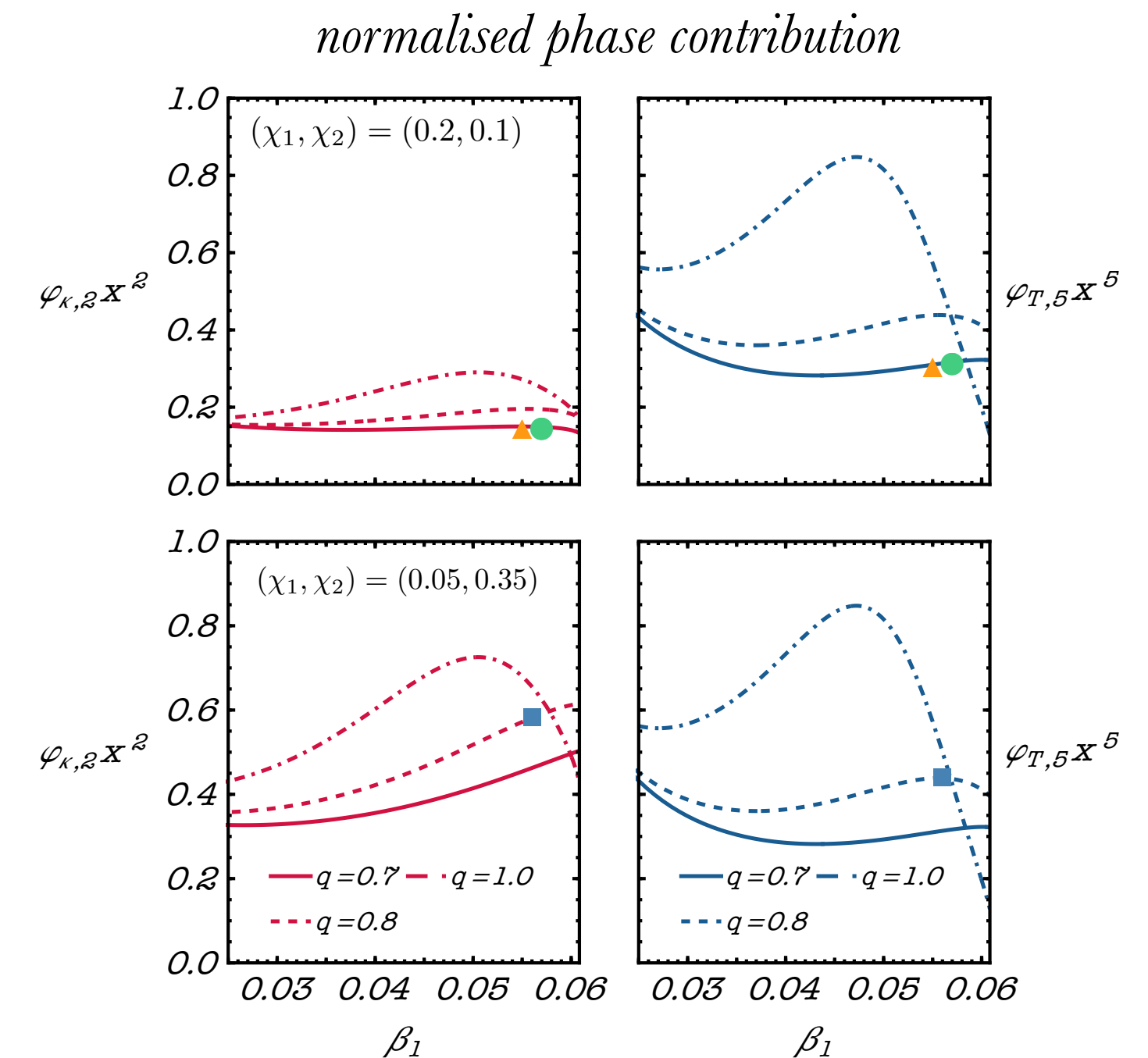
A coherent waveform model

Bayesian study for 3g detectors (Einstein Telescope, Cosmic Explorer) [M. Vaglio + inc. A.M, 2023]

- Injection/recovery using inspiral only signals (TaylorF2) with different configurations
- Sampling on the waveform parameters up to mass shedding ($q = m_2/m_1 \leq 1$)

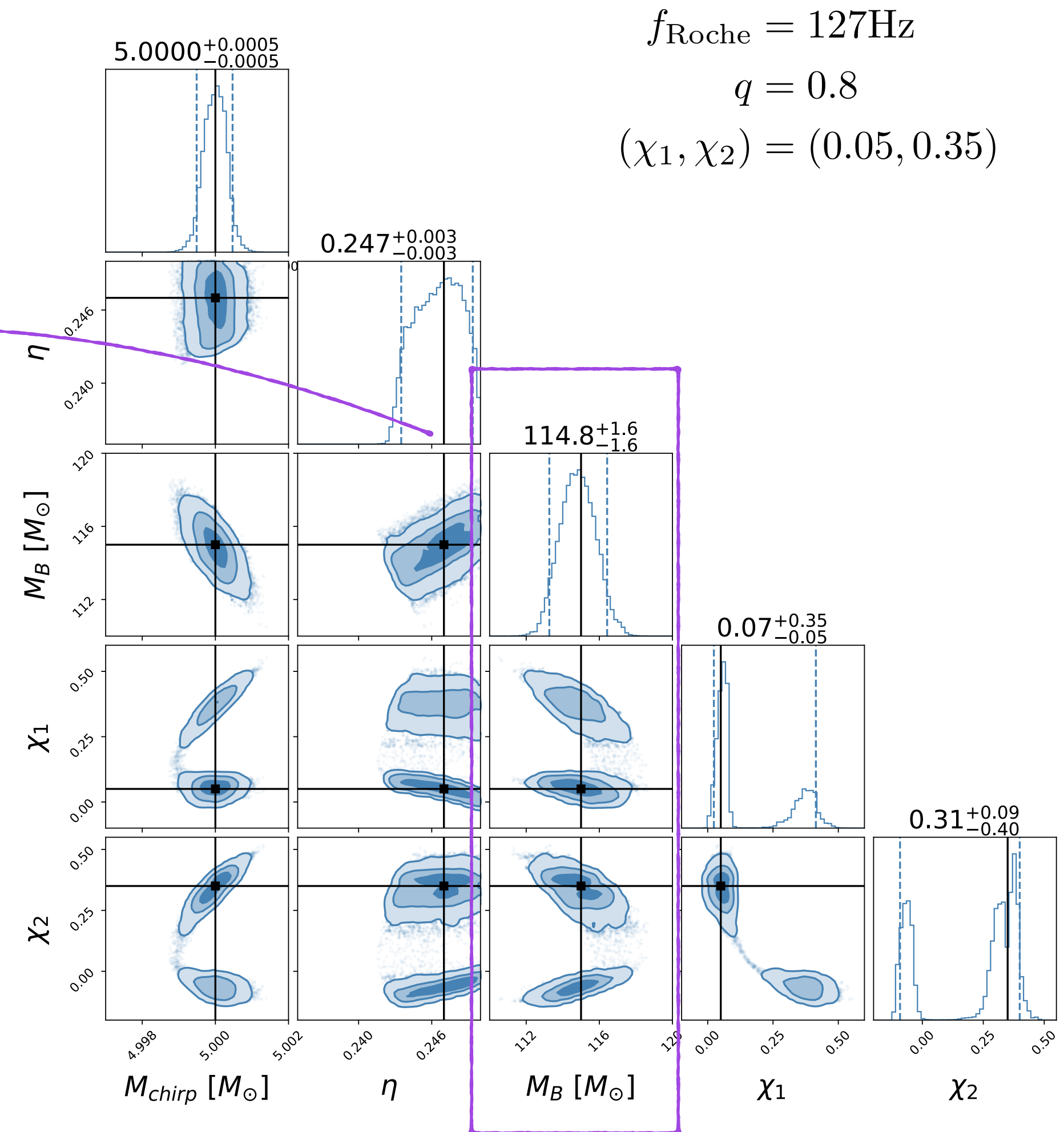
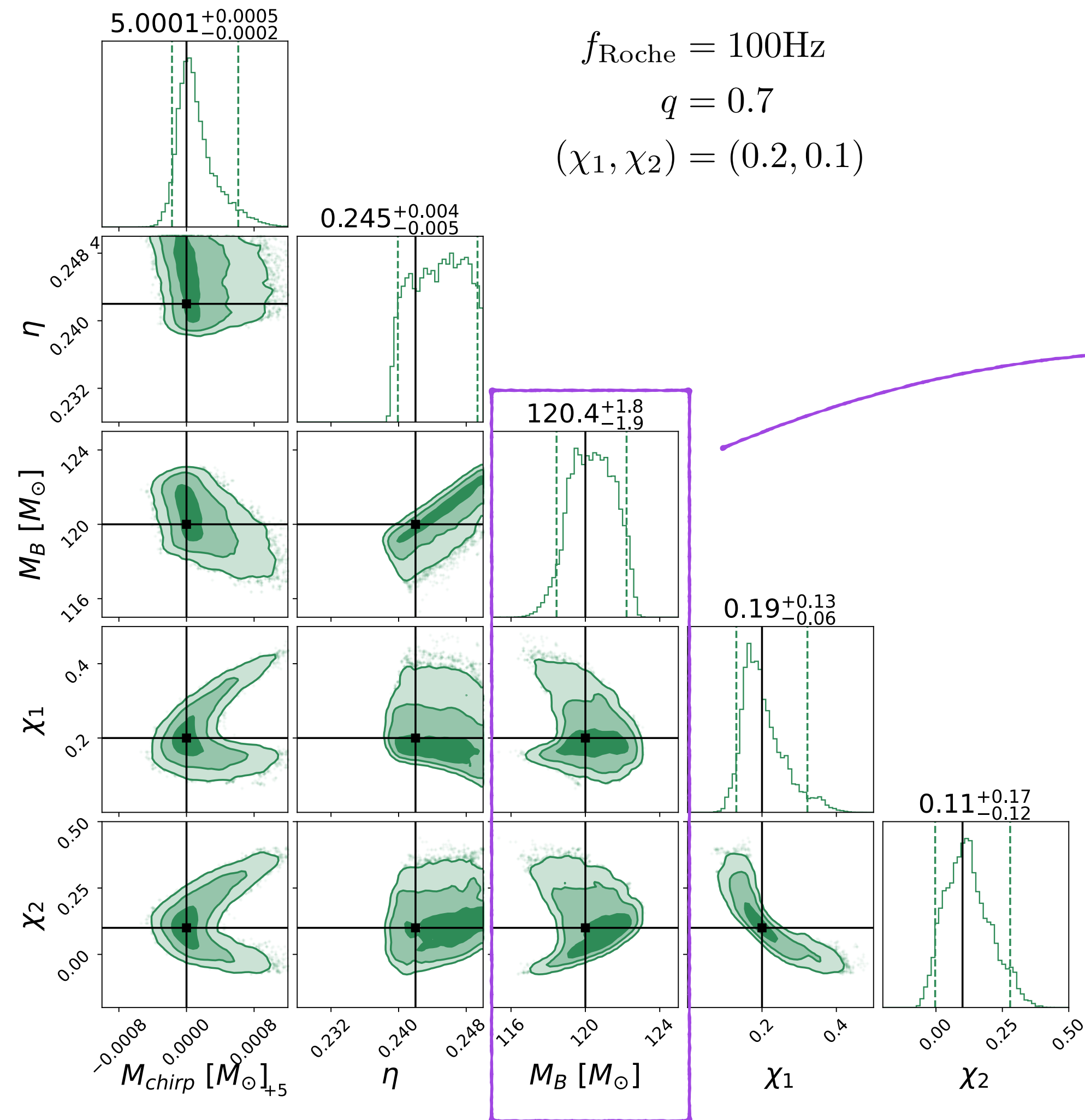
$$r_{\text{Roche}} \sim \gamma r_2 \left(\frac{m_1}{m_2} \right)^{\frac{1}{3}} \longrightarrow f_{\text{Roche}} = \frac{1}{\pi m} \sqrt{3 + q + 3q^{-1} + q^{-2}} \left(\frac{\mathcal{C}_2}{\gamma} \right)^{\frac{3}{2}}$$

$(m_1, m_2) [M_\odot]$	(β_1, β_2)	η	$M_B [M_\odot]$	χ_1	χ_2	
(6.9, 4.8)	(0.057, 0.040)	0.242	120	0.20	0.10	●
(6.4, 5.2)	(0.056, 0.045)	0.247	115	0.05	0.35	■
(13.8, 9.6)	(0.055, 0.039)	0.242	250	0.20	0.10	▲



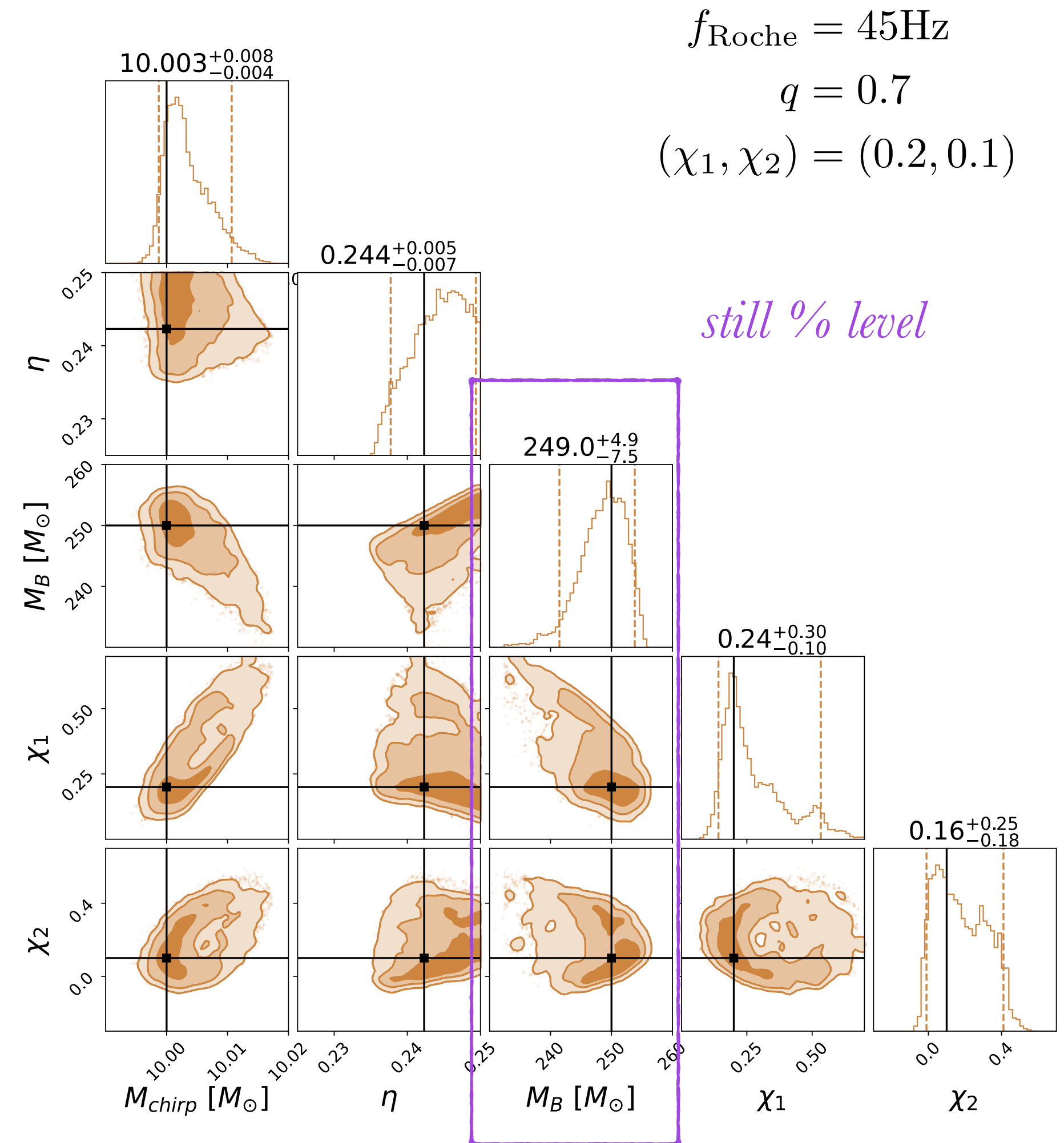
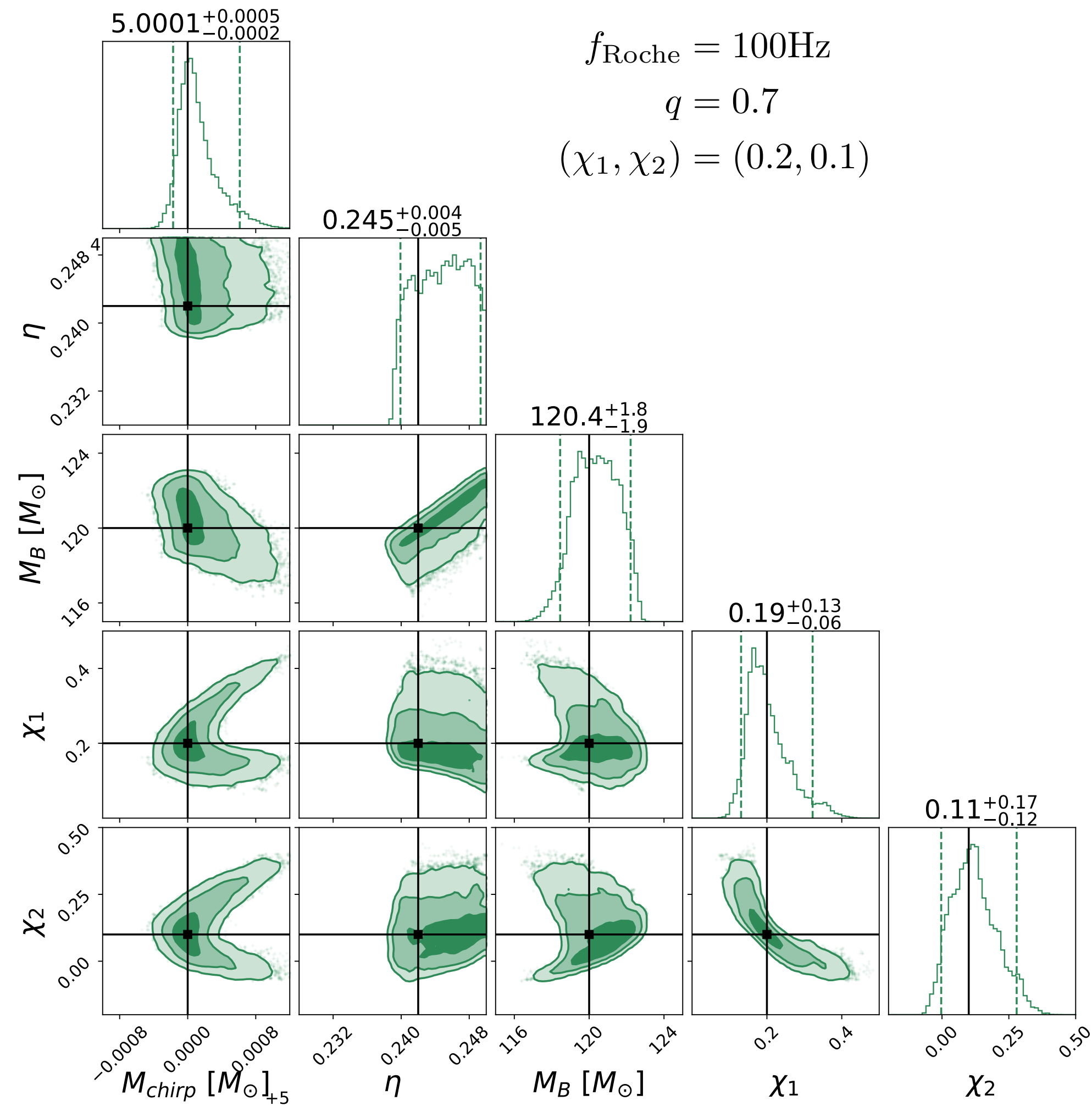
A coherent waveform model

Reconstruction of the source parameters



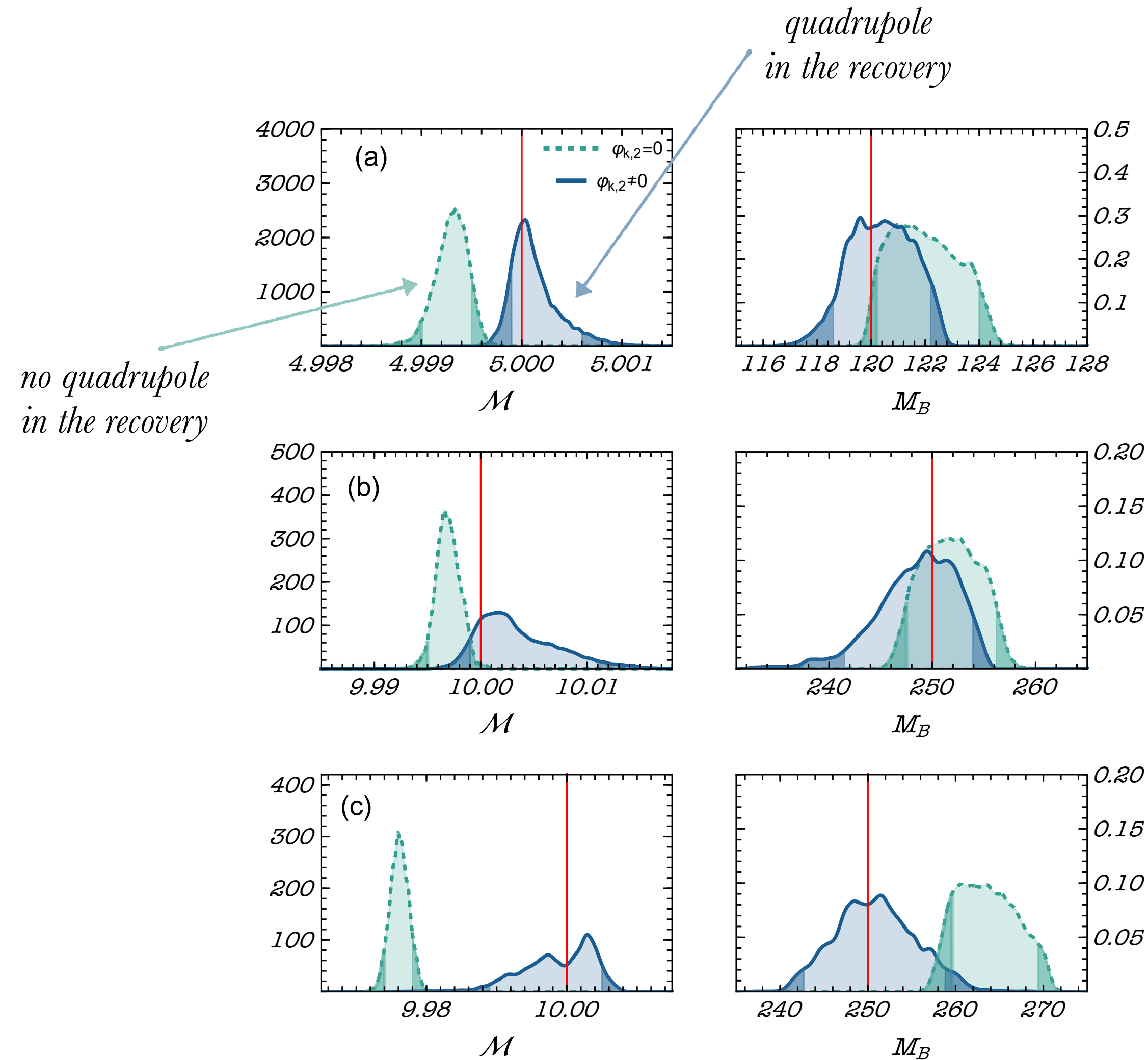
A coherent waveform model

Reconstruction of the source parameters



A coherent waveform model

Including the quadrupole matters



Speaking of the bias

Is there any systematic error in the source parameters when using a wrong waveforms model?

[E. Berti + inc. A.M., 2025]

- Equal mass binaries observed by ET & LISA

Injection: GW signal with $\tilde{\Lambda} \neq 0$

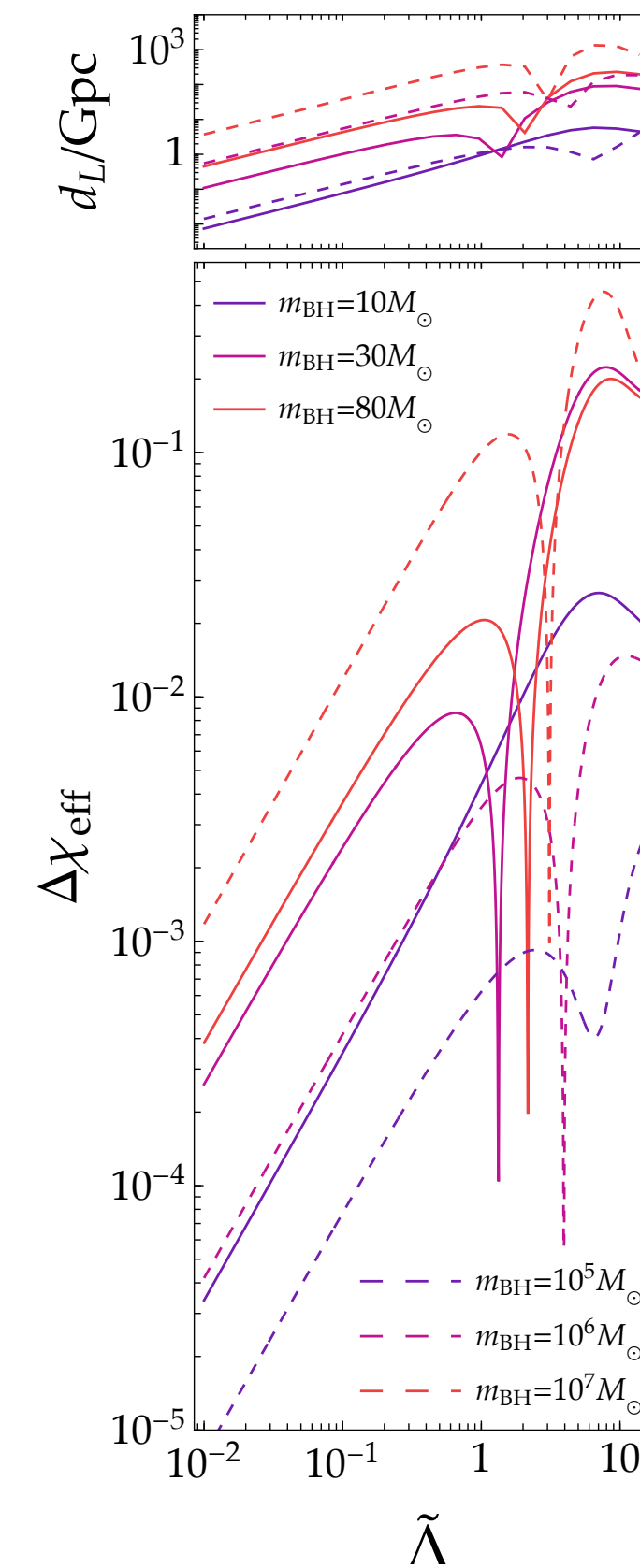
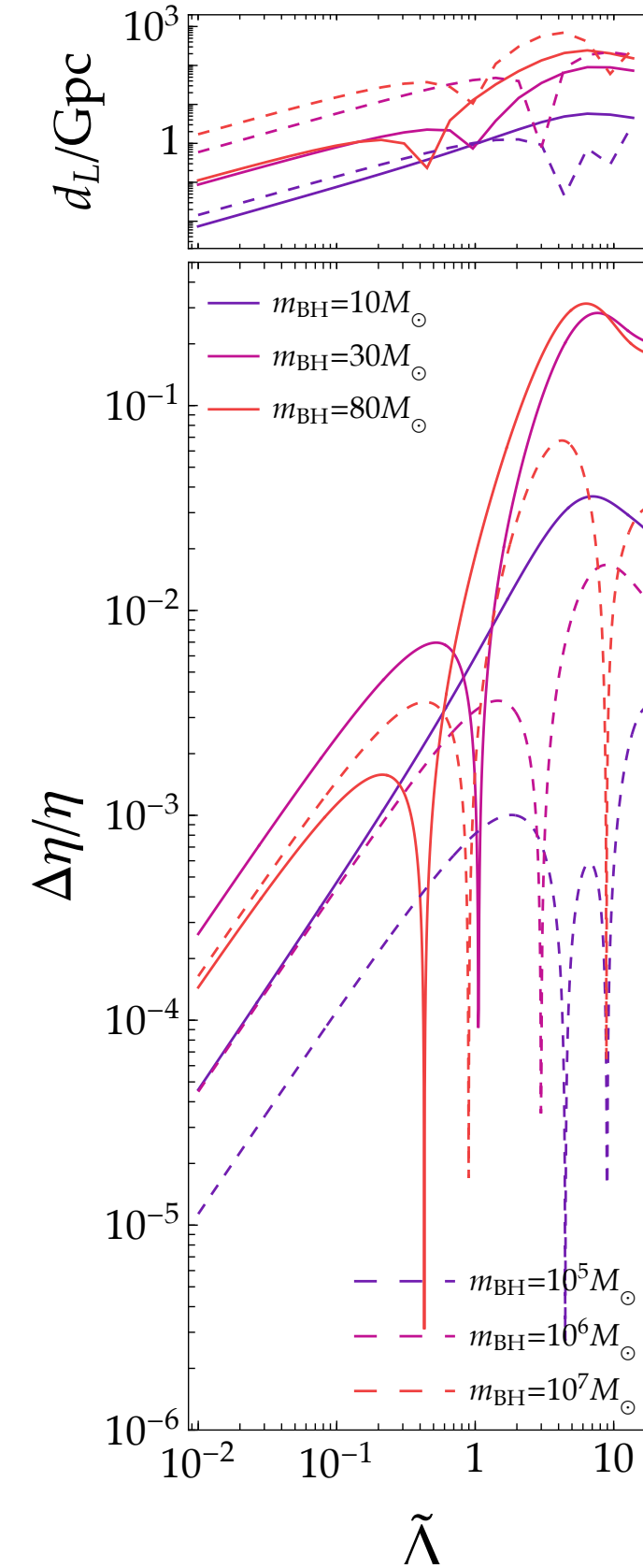
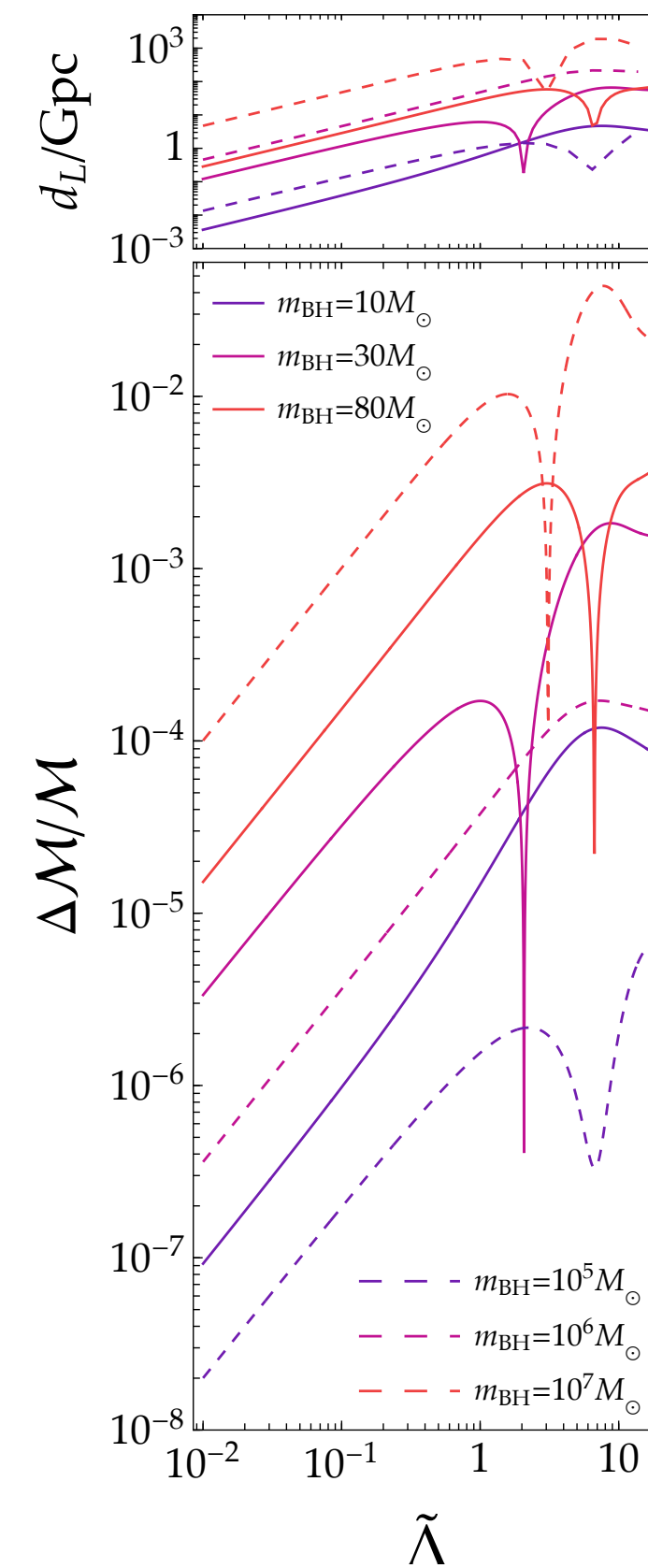


Recovery: GW signal with $\tilde{\Lambda} = 0$

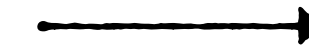


$$\Delta\theta \propto \Lambda$$

*bias over
injected value*



*distance @
bias $\gtrsim$ statistical error*



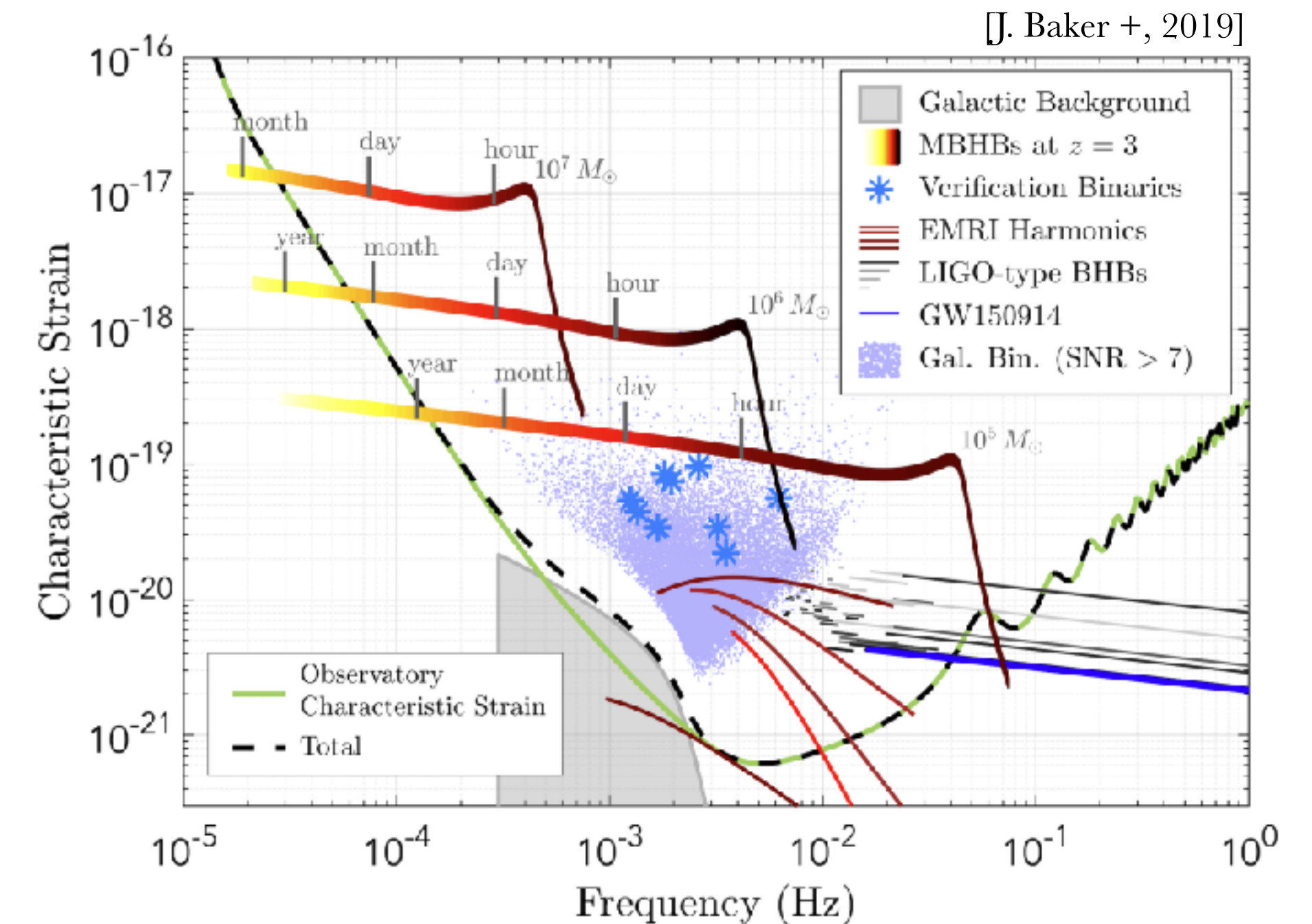
Asymmetric binaries and next gen detectors

90+ events observed so far from LVK, spanning a relatively small interval of mass ratios $q \sim 1 : 30$

- Space and ground based detectors can beat down such value by several orders of magnitudes $m/M = q \sim 10^{-2} - 10^{-7}$
- Dynamics dictated by q , with the duration of the inspiral & number of cycles growing as q decreases
- E/IMRIs as golden sources for milli-Hz and deci-Hz detectors

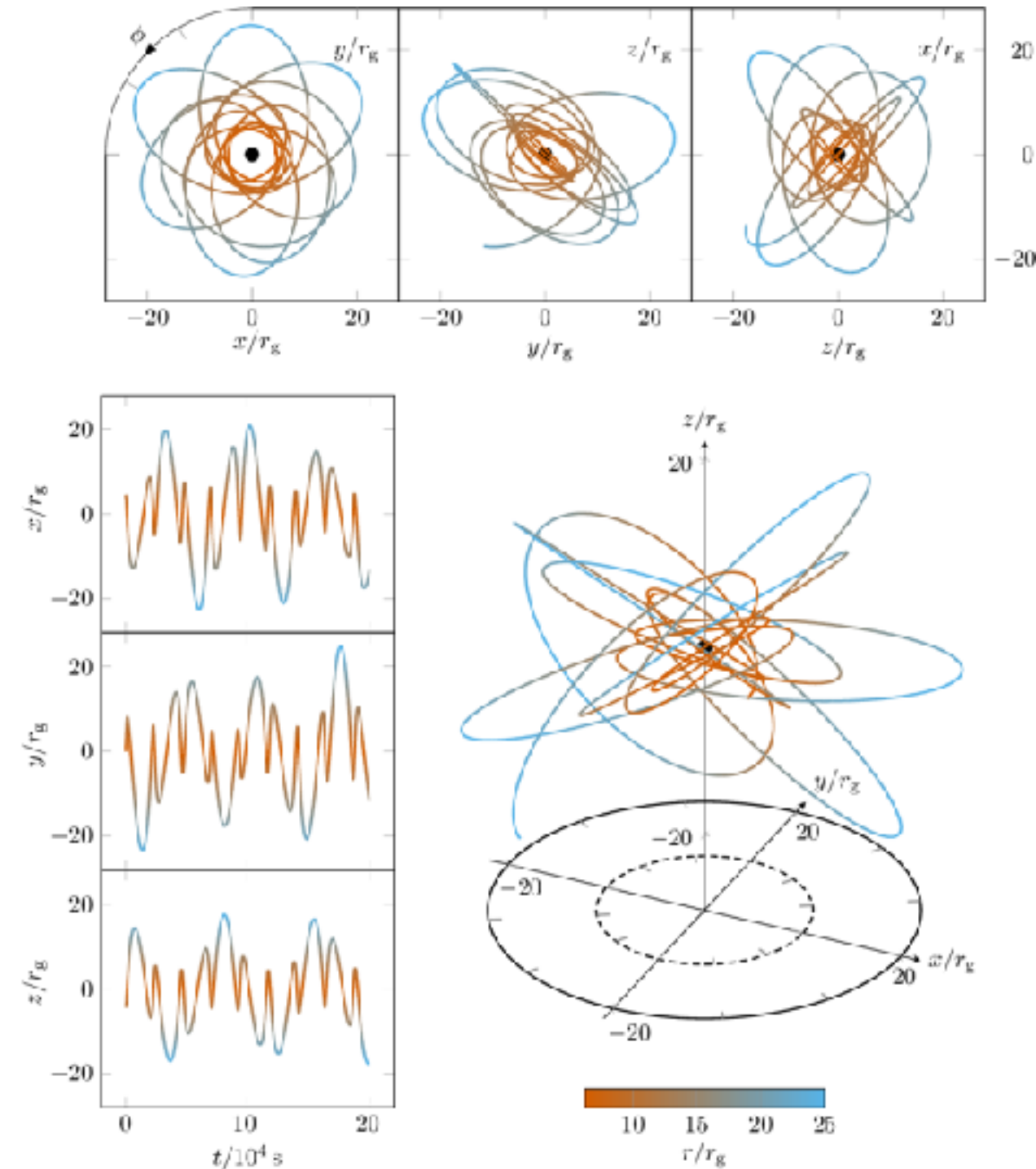
Discovery potential

- Slow inspiral phase which could allow to continuously observe EMRI/IMRI for very long periods, from months to years
- Dynamical evolutions with an uncommon richness, with resonances, large eccentricities and off-equatorial orbits, etc.
- rich astro-fundamental physics science cases



EMRIs

EMRIs provide a rich phenomenology, due to their orbital features



Berry +, Astro2020 1903.03686 (2019)

- Non equatorial orbits
- Eccentric motion
- Resonances
- Complete $\sim 10^4 - 10^5$ cycles before the plunge

blessing in disguise

Tracking EMRIs for $O(\text{year})$ requires accurate templates

Precise space-time map and accurate binary parameters

Very appealing to test fundamental & astro-physics [A. Avendāno & C. Sopuerta, 2024]

- How do we include and test *new physics* with such sources?

EMRIs in nuce

The asymmetric character introduces a natural parameter to study the problem in perturbation theory $q = m_p/M \ll 1$

$$g_{\alpha\beta} = g_{\alpha\beta}^{(0)} + h_{\alpha\beta} + \dots$$

$$G_{\mu\nu} = T_{\mu\nu}^p = 8\pi m_p \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy_p^\alpha}{d\lambda} \frac{dy_p^\beta}{d\lambda} d\lambda$$

leading
adiabatic

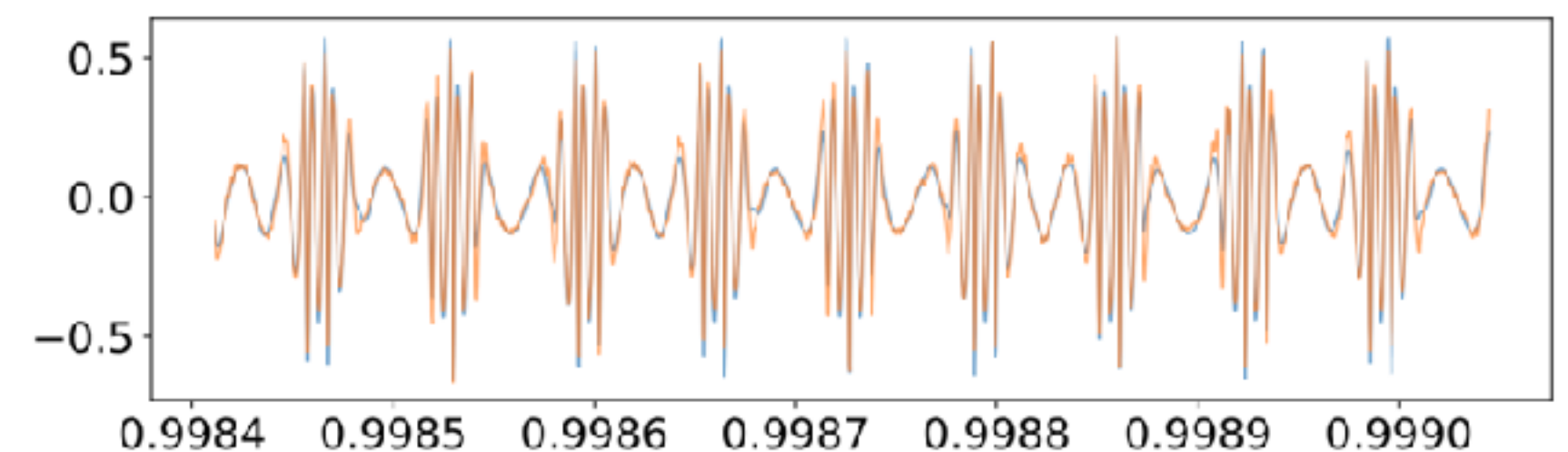
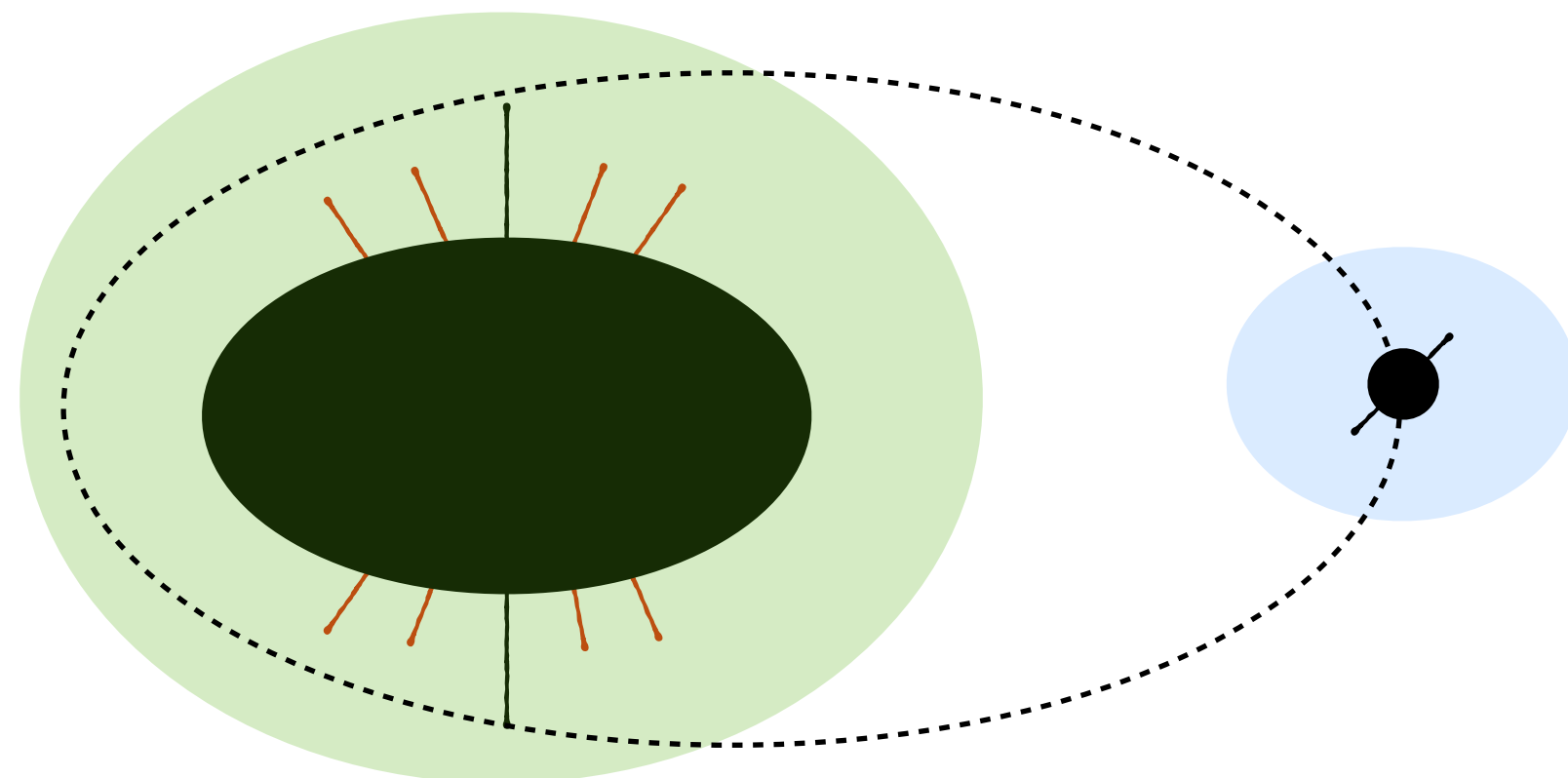
Regge-Wheeler-Zerilli
(Schwarzschild)

Teukolsky
(Kerr)

○ The solution determines the phase evolution

$$\phi(t) = \overset{\text{adiabatic}}{\phi_{\text{diss-1}}} + \overset{\text{first post-adiabatic}}{\dots}$$

$\downarrow \mathcal{O}(1/q)$
 $\downarrow \mathcal{O}(1)$



EMRIs and tidal effects

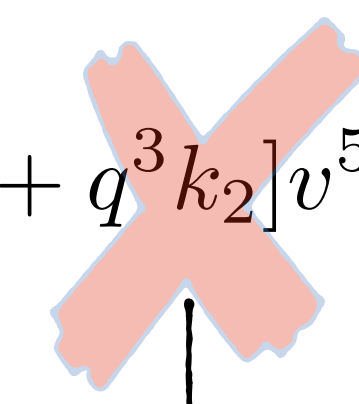
Multipolar structure and absence of the horizon as signatures of non-Kerrness for the massive component

[F. Ryan, 1997, G. Raposo +, 2019, C. Herdeiro +, 2021, N. Loutrel +, 2022, K. Fransen & D. Mayerson 2022]

Tidal effects for asymmetric binaries. Learning some lesson from pN expansion (EMRIs are not pN binaries though....)

- Scaling of the pN tidal phase for binaries with $q \ll 1$.

$$\phi_{\text{tidal}}(f) \propto [k_1 q^{-1} + q^3 k_2] v^5$$



Love number primary

Love number secondary

- Tidal deformability of the primary is naturally “promoted” within the phase hierarchy
- Same order in q of the leading point-particle contribution $\phi_N(f) \propto v^{-5} q^{-1}$

- with $k_1 \gg q$ tidal effects become more important than conservative effects which are $\mathcal{O}(q)$

[P. Pani, A. M., 2019]

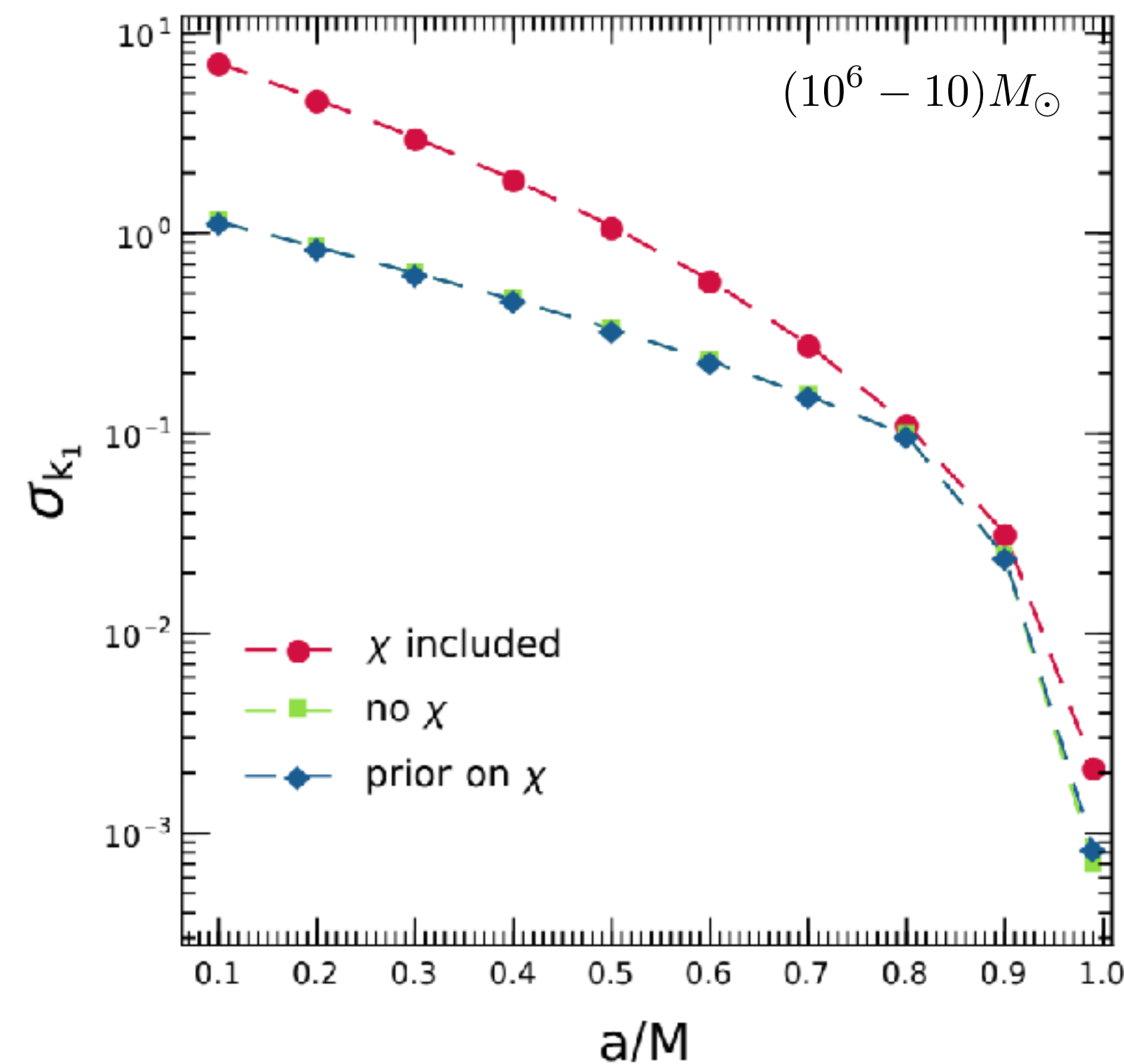
EMRIs and tidal effects

[G. Piovano, A. M., P. Pani 2023]

Uncertainties on the tidal Love number from EMRI observations by LISA

Binary evolution with “kludge” waveforms

Relativistic amplitudes and mixed GW fluxes $\dot{\mathcal{E}} = \dot{\mathcal{E}}^{\text{Teuk}} + \dot{\mathcal{E}}_{\text{pN}}^{\text{tidal}}$



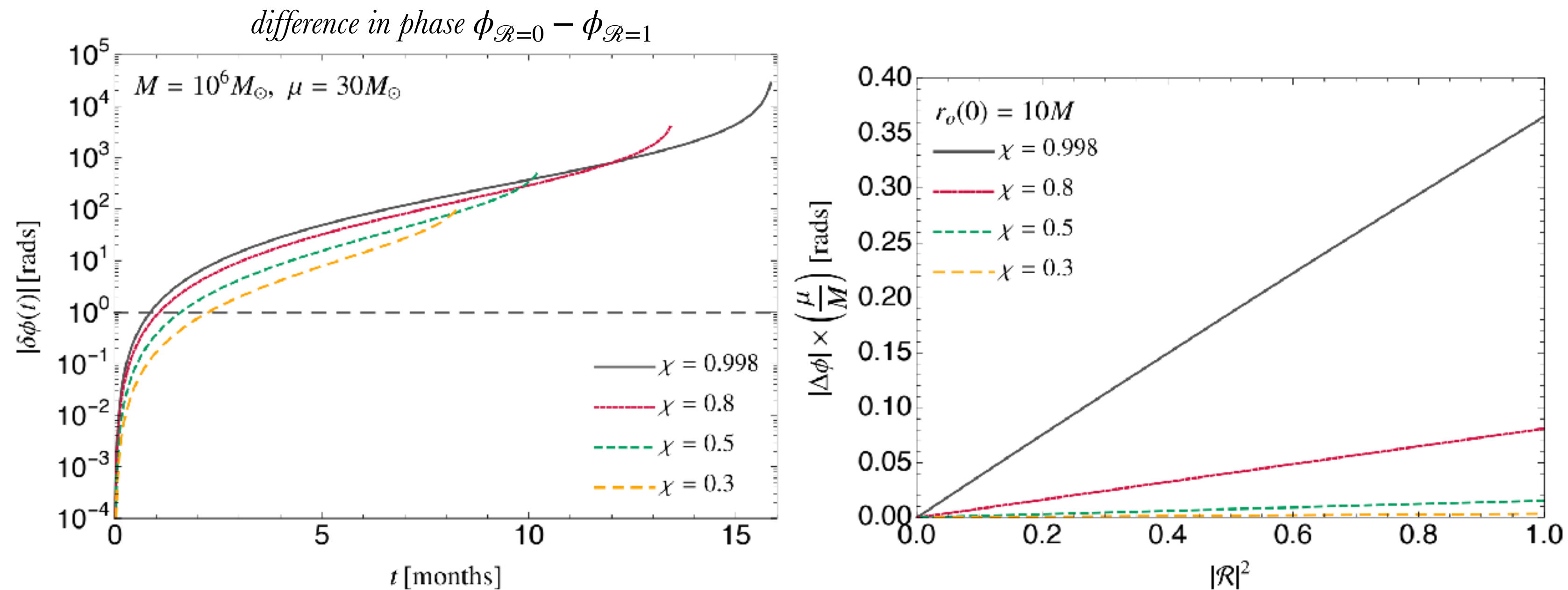
*As reference, $\sigma_{k_1} \lesssim 10^3$
for constraints on tidal effects from
GW170817*

EMRIs and heating

Absence of horizon as potential discriminator for non - Kerrness

● Boundary conditions, QNMs, low frequency modes, partial absorption [S. Hughes 2001; S Bernuzzi +, 2012; P. Pani +, 2009; E. Maggio +, 2017; E. Maggio +, 2017; P. Pani +, 2010; C. Macedo +, 2013, A. M. +, 2018; V. Cardoso +, 2019]

● (partial) reflection by surface $\dot{\mathcal{E}}^{\text{surf}} = (1 - |\mathcal{R}|^2)\dot{\mathcal{E}}^H$ [S. Datta +, 2020]



EMRIs and heating

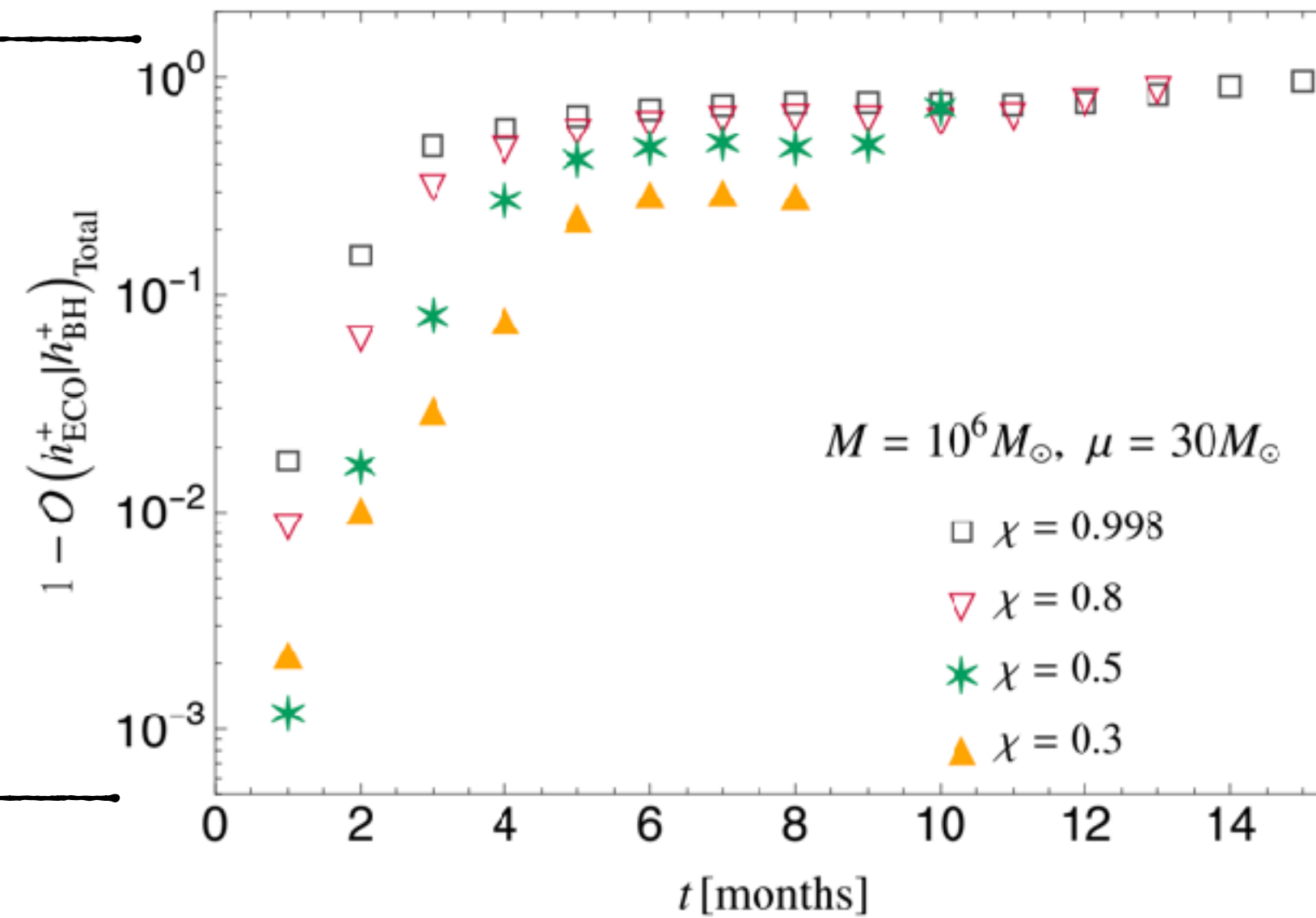
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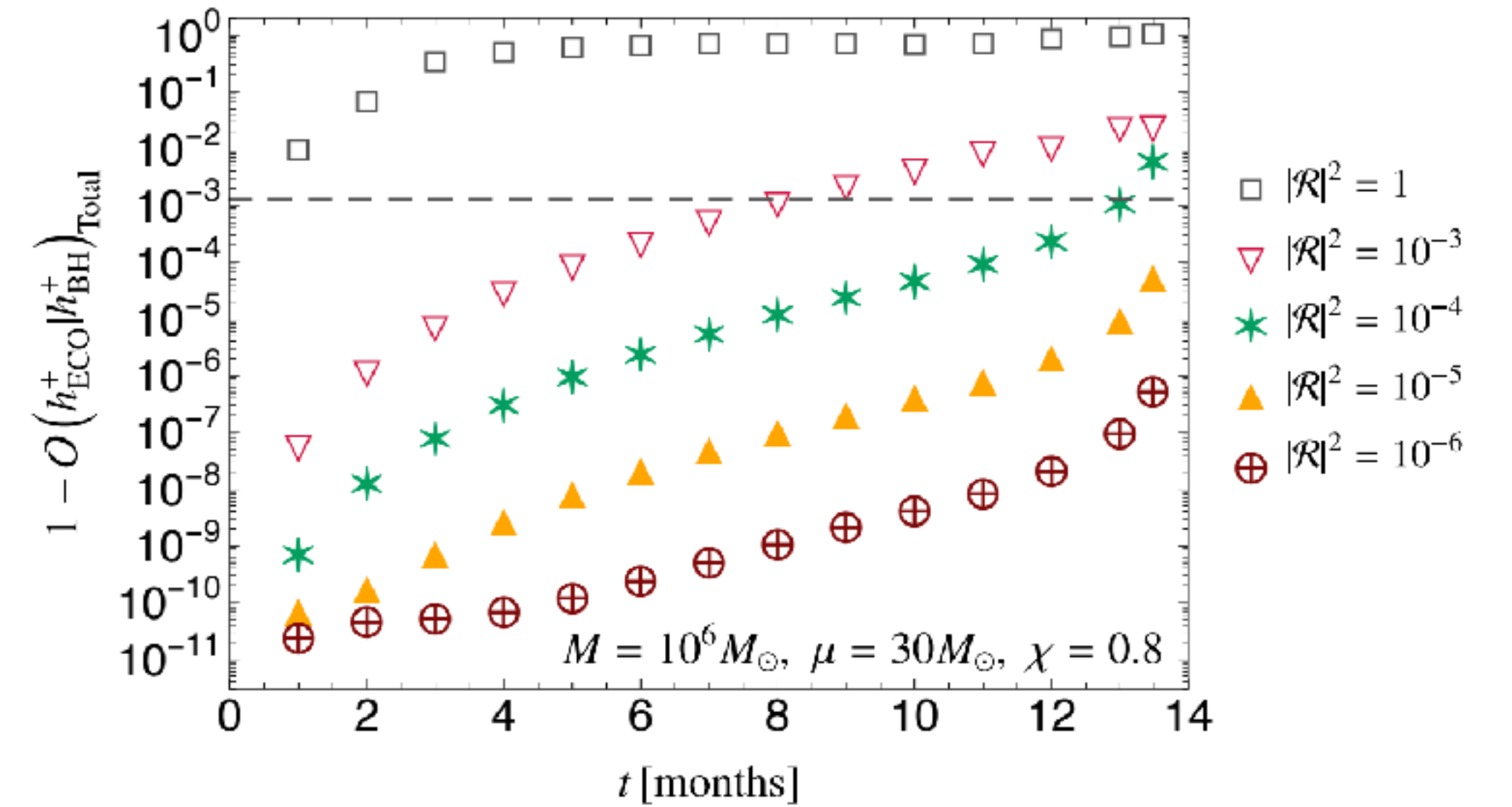
- (partial) reflection by surface $\dot{\mathcal{E}}^{\text{surf}} = (1 - |\mathcal{R}|^2)\dot{\mathcal{E}}^H$ [S. Datta +, 2020]

$$\mathcal{O}(h_1|h_2) = \max_{t_0, \phi_{i0}} \frac{\langle h_1|h_2 \rangle}{\sqrt{\langle h_1|h_1 \rangle \langle h_2|h_2 \rangle}} \xrightarrow{\text{weighted by LISA sensitivity}}$$

very different



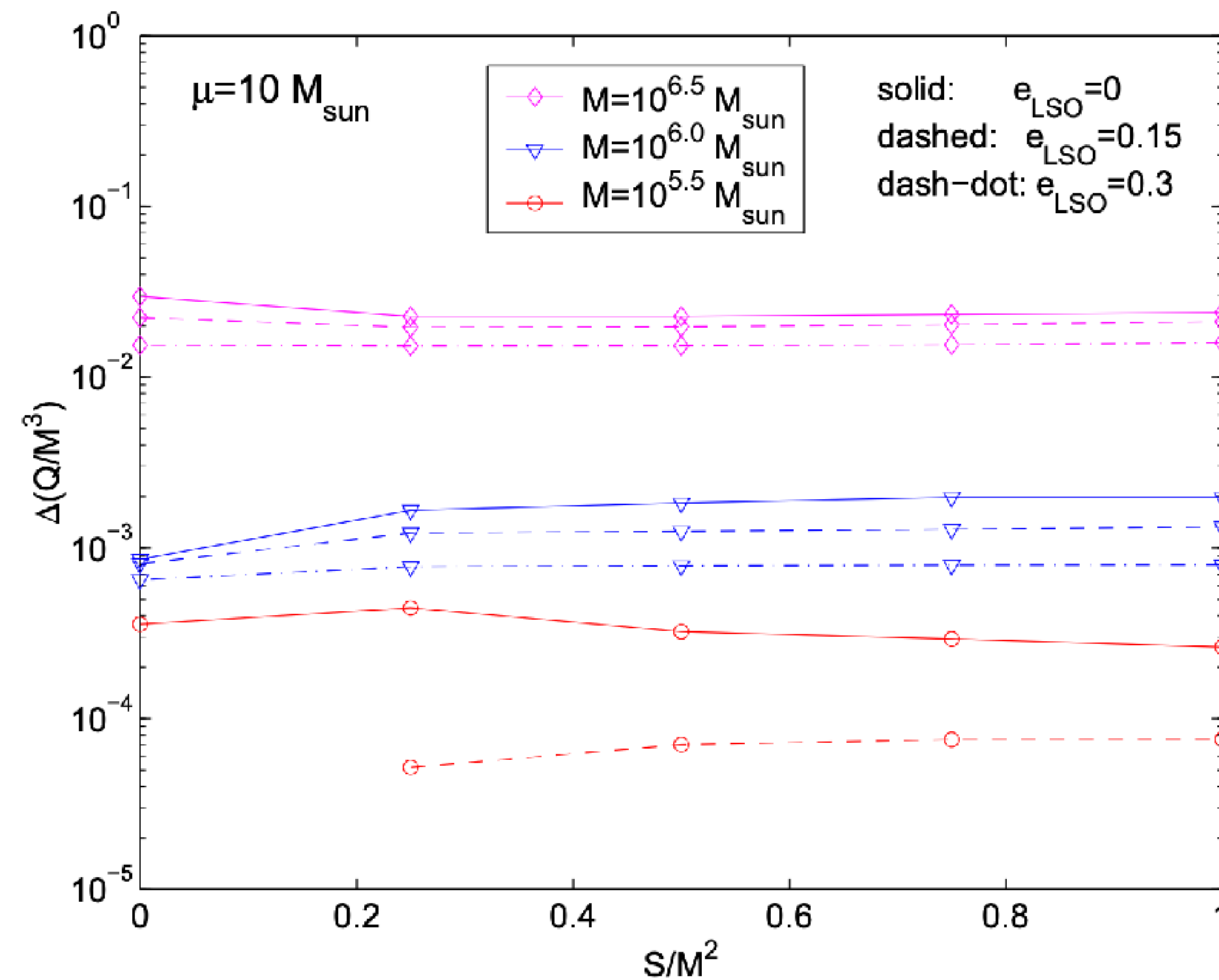
the same



Other smoking guns? Quadrupole moment

Measuring the quadrupole moment of the massive EMRI component with LISA observations

[L. Barack & C. Cutler, 2006]



Thoughts

Using the inspiral phase of binary merger looks an appealing environment for searching new physics

- 🟢 “recycle” theoretical developments, techniques, data analysis approaches...
- 🔴 Do we have a faithful model for fundamental physics?

pN approaches

- 🕒 Installed on BH/NS baseline models (theory still can and needs to inform the way we construct them)
- 🕒 Goal to construct hybrid models. Few examples on realistic models [T. Evstafyeva + 2024; N. Siemonsen 2024; N. Siemonsen & W. East, 2023]
- 🕒 ok for early (?) inspiral. When do we stop?

SF approaches

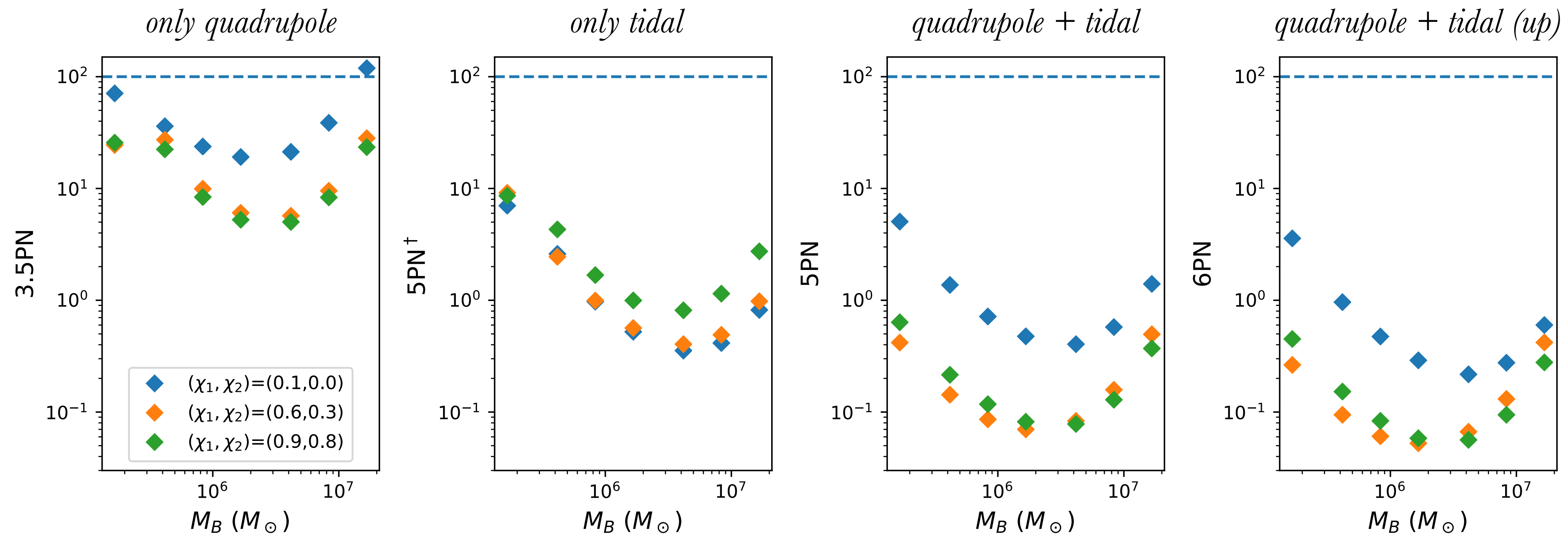
- 🕒 Vast majority of works/analysis exploratory science
- 🕒 At very best “kludge” waveform models mixing fully relativistic & pN results

Back up

A coherent waveform model

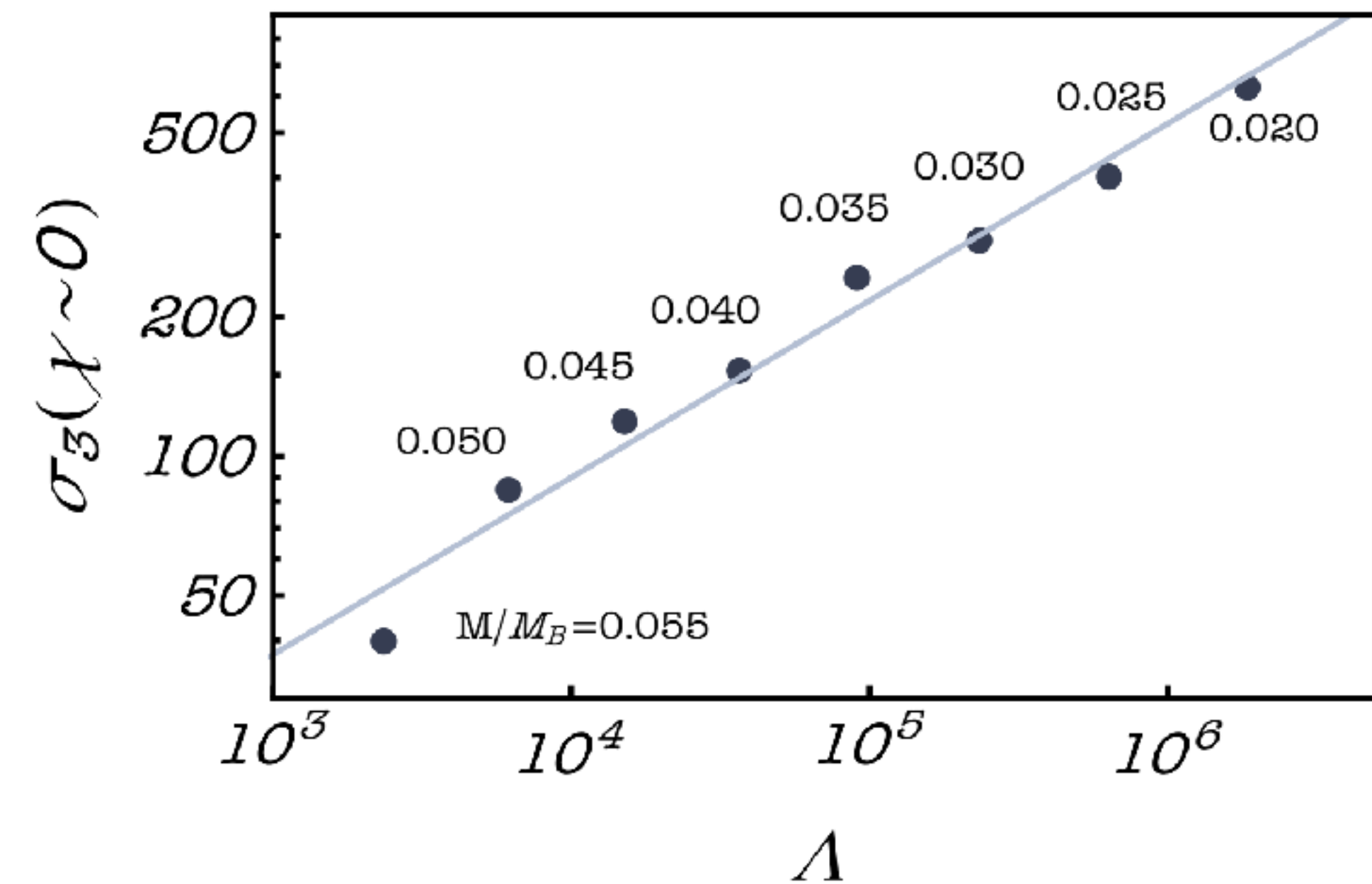
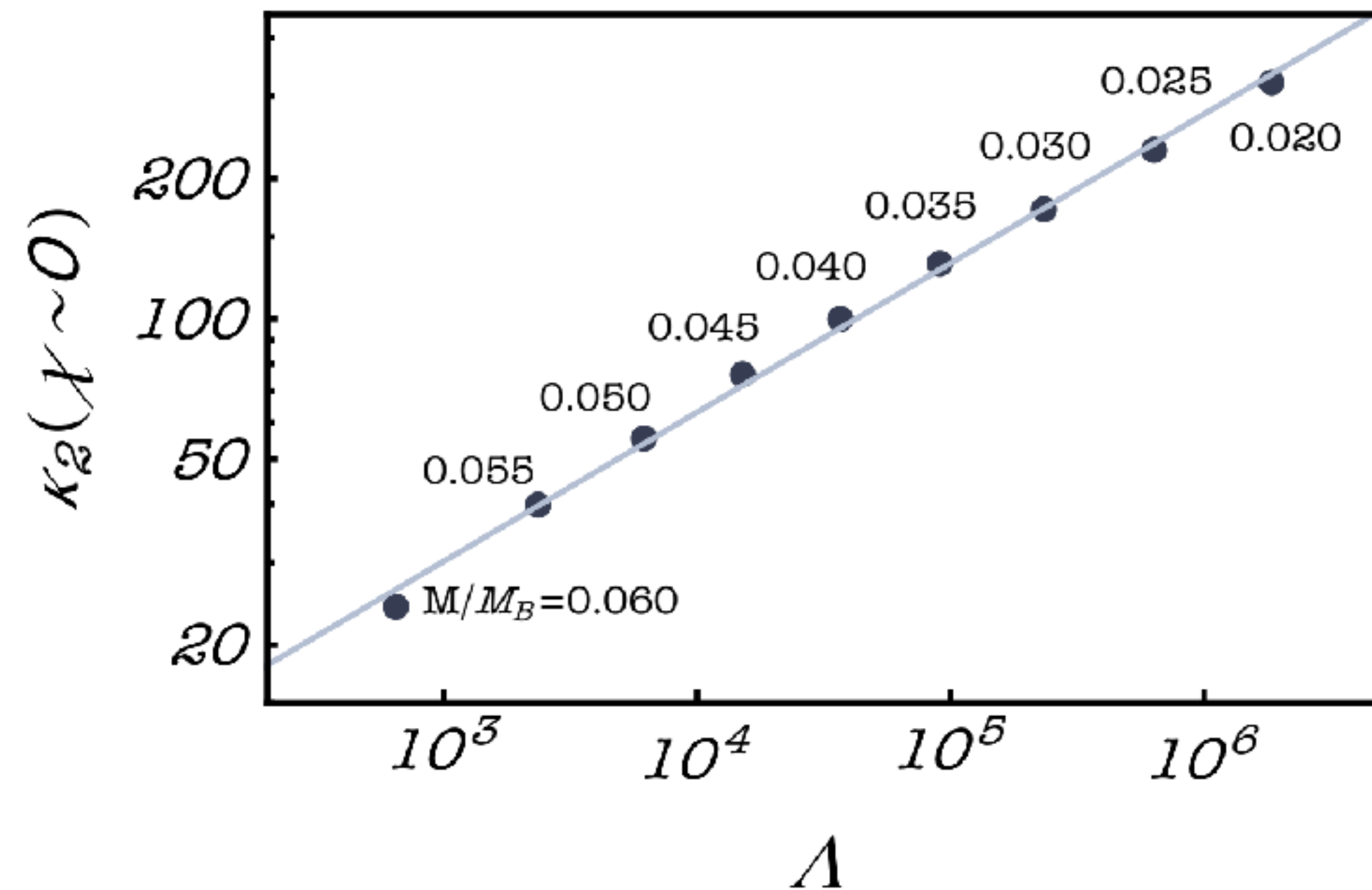
Constraining M_B with massive binaries evolving in the LISA band

- Fisher matrix analysis including different phase contributions



Universal relations among multipole moments

Semi-analytic relation between mass quadrupole and mass octupole and tidal deformability [M. Vaglio + inc. A.M, 2022]



- Can be used to break degeneracy among the source parameters

EMRIs in nuce

$$\mathbf{g}_{\alpha\beta} = g_{\alpha\beta} + qh_{\alpha\beta} + q^2 h_{\alpha\beta}^{(2)} + \mathcal{O}(q^3)$$

Contributions to the orbital trajectory

$$\frac{D^2 z^\alpha}{d\tau^2} = qf_1^\alpha + q^2 f_2^\alpha + \mathcal{O}(q^3)$$

Inspiral evolution on radiation-reaction time t_{rr}

$$t_{rr} = \mathcal{E}/\dot{\mathcal{E}} \sim M/q \quad \xrightarrow[\text{of second order SF}]{\text{cumulative shift}} \quad \delta z^\alpha \sim q^2 f_2^\alpha t_{rr}^2 \sim q^0$$

Match filtering require error in phase $\ll 1$ radian: f_2^α  f_3^α 

$$\Phi(t) = \frac{1}{q} \left[\Phi_0(t) + q\Phi_1(t) + \mathcal{O}(q^3) \right]$$

$f_{1,\text{diss}}^\alpha$

$f_{1,\text{cons}}^\alpha$
 $f_{2,\text{diss}}^\alpha$

[T. Hinderer & E. Flanagan, 2008]